

# Viterbi Decoder Node Synchronization Losses in the Reed-Solomon/Viterbi Concatenated Channel

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*The Viterbi decoders currently used by the Deep Space Network (DSN) employ an algorithm for maintaining node synchronization that significantly degrades at bit signal-to-noise ratios (SNRs) of below 2.0 dB. In a recent report by the authors, it was shown that the telemetry receiving system, which uses a convolutionally encoded downlink, will suffer losses of 0.85 dB and 1.25 dB respectively at Voyager 2 Uranus and Neptune encounters. This report extends the results of that study to a concatenated (255, 223) Reed-Solomon/(7, 1/2) convolutionally coded channel, by developing a new radio loss model for the concatenated channel. It is shown here that losses due to improper node synchronization of 0.57 dB at Uranus and 1.0 dB at Neptune can be expected if concatenated coding is used along with an array of one 64-meter and three 34-meter antennas.*

## I. Introduction

All planned NASA and European Space Agency (ESA) deep space missions will have the capability of using a concatenated Reed-Solomon/convolutional coding scheme for downlink telemetry (Ref. 1). Voyager 2 also has this capability on board, and its encounter with Uranus in 1986 will be the first use of this scheme by a space flight project. A simplified block diagram of a concatenated coded downlink system is shown in Fig. 1.

Although the specific details of the codes may differ among the various missions, the basic code parameters, and hence the overall system performances, are identical. The convolutional inner code is a  $k = 7$ , rate  $1/2$  code. The Reed-Solomon

outer code is a (255, 223) code with 8-bit symbols. The differences in the codes for specific missions involve different orderings and inversions in the convolutional code's connection vectors and different finite field representations and generating polynomials for the Reed-Solomon code. The baseline performance of this coding scene in the presence of space loss and receiver thermal noise was determined in Ref. 2. It is the purpose of this report to model two additional losses. These are noisy carrier referencing (or "radio loss") and imperfect Viterbi decoder node synchronization. These losses were treated for the convolutional-only channel in Ref. 3.

Noisy carrier referencing is a degradation caused by the effects of noise on the carrier tracking loss in the receiver

(Ref. 4). The noise causes the phase-locked loop to incorrectly estimate the phase of the incoming signal, which results in a nonoptimal signal demodulation. The traditional radio loss models do not apply in the case of concatenated coding, since carrier phase tracking errors vary slowly compared to the decision times in the Viterbi decoder but quickly compared to the decision times in the Reed-Solomon decoder. Thus it was necessary to develop a new model in Section II herein called the mixed-rate model.

The node synchronization loss is a degradation caused by the Viterbi decoder. The encoder for the (7, 1/2) convolutional code outputs a pair of channel symbols for each input bit. Conversely, the decoder must parse the received symbol stream correctly to synchronize the symbols into pairs. When the Viterbi decoder parses the data incorrectly, it is said to be out of node synchronization. The current DSN Viterbi decoders use an internal algorithm for maintaining node synchronization that fails below a certain channel SNR. This critical SNR value is called the node synchronization threshold. Hardware tests (Ref. 5) and mathematical modeling (Ref. 3) have demonstrated that the node synchronization threshold of the Maximum-Likelihood Convolutional Decoders (the DSN's Viterbi decoders, or MCDs) is about 2.0 dB.

The modeling described in Sections II and III of this report predicts that losses to the concatenated coding system due to poor node synchronization will amount to 0.57 dB at Voyager 2 Uranus encounter and 1.0 dB at Neptune encounter. These results assume that a perfect carrier array consisting of one 64-meter antenna and three 34-meter antennas is used for telemetry reception.

## II. The Mixed-Rate Model

This section describes the model used to calculate the effects of noisy carrier referencing on concatenated coded telemetry. Traditionally, there have been two basic models used for radio loss calculations for coded data: the "high-rate model" and the "low-rate model." The high-rate model assumes that the length of time needed by the decoder for decisions is so small that the phase estimate in the carrier tracking loop is constant. The decoded bit error rate performance is therefore given by the average of the bit error rates caused by the different possible phase errors. This is reflected in the equation

$$p_{bit}(x) = \int_{-\pi}^{\pi} f(x \cos^2 \phi) p(\phi) d\phi. \quad (1)$$

In equation (1),  $x$  is the bit SNR incident upon the receiving antenna. The effect of a phase error  $\phi$  is a degradation of

$\cos^2 \phi$  in SNR. The quantity  $p(\phi)$  is the phase error density function and  $f(x)$  is the decoder's bit error rate as a function of SNR in the case of perfect carrier tracking.

The low-rate model, on the other hand, assumes that the phase errors change so much faster than the time it takes to decode one bit that an average SNR can be used. This leads to the equation

$$p_{bit}(x) = f \left[ x \int_{-\pi}^{\pi} \cos^2 \phi p(\phi) d\phi \right] \quad (2)$$

For data rates and carrier loop bandwidths that are typical of Voyager planetary encounters, carrier phase errors remain relatively constant for 600 data bit times (Ref. 3). Since the memory length of the Viterbi decoders is only 64 bits, the high-rate model applies to the convolutional-only channel. Each Reed-Solomon word, however, is 2040 bits in length. In addition, interleaving can further delay the decoding time, so that the low-rate model applies to a Reed-Solomon-only channel. Unfortunately, neither model applies well to the concatenated channel.

The mixed-rate model resolves this dilemma by applying the high-rate model to the inner convolutional code and the low-rate model to the outer Reed-Solomon code. First, Eq. (1) is used to calculate the Viterbi-decoded bit error probability of the inner convolutional channel. The value  $x$  is taken to be  $E_v/N_0$ , the bit SNR of the inner convolutional channel. The function  $f(x)$  is taken to be the ideal (no radio loss) performance of the Viterbi decoder. The phase error density  $p(\phi)$  is derived in Ref. 5 to be

$$p(\phi) = \frac{\exp(\cos \phi)}{2\pi I_0(\rho)}$$

where  $\rho$  is the loop SNR of the carrier tracking loop, and  $I_0$  is the zero order modified Bessel function.

The average Reed-Solomon symbol error rate (the probability of one or more errors occurring in a string of eight consecutive bits)  $\pi$  is estimated to be  $2.5 p_{bit}$ , where  $p_{bit}$  is the bit error rate of the inner convolutional channel. Figure 2 shows the ratio  $\pi/p_{bit}$  for various channel and carrier loop SNRs. A ratio of 2.5 is about average for SNRs in the 2-3 dB range of  $E_v/N_0$ , where most of the mass of the integral in (1) occurs. The overall concatenated bit error rate is then calculated assuming that the inner channel bit error rate  $p_{bit}$  and the symbol error rate  $\pi$  vary quickly with respect to a Reed-Solomon decoder decision time. The mixed-rate model yields a concatenated bit error rate of

$$p_{RS} = \frac{p_{bit}}{\pi} \sum_{i=17}^{255} \binom{255}{i} \left(\frac{i}{255}\right) \pi^i (1-\pi)^{255-i}. \quad (3)$$

The bit SNR of the concatenated channel is taken to be

$$E_b/N_0 = (E_v/N_0) 255/223$$

due to the additional overhead of the Reed-Solomon code.

In the case of infinite carrier loop SNR, i.e., no radio loss,  $p_{bit} = f(E_v/N_0)$ . Figure 3 shows  $p_{RS}$  as a function of  $E_b/N_0$  for this case with the assumption that  $\pi/p_{bit}$  is 2, 2.5 or 3. The total difference between the performances predicted by these hypotheses at the bit error rate of 10 is less than 0.17 dB. This implies that the assumption that this ratio is a constant will not seriously affect the relative performance estimates made by using the mixed-rate model.

### III. Node Synchronization Model

The model of Viterbi decoder node synchronization losses used in this study is described in Ref. 3. It is assumed that for values of  $E_v/N_0$  above the node synchronization threshold the Viterbi decoder performs normally and in perfect synchronization. For SNRs below the threshold, the decoder's internal synchronization algorithm always believes that the decoder is out of node synchronization. The result of this condition is that the decoder continually oscillates between correct and incorrect node synchronization and produces an essentially random output.

In order to introduce the above information into the mixed-rate model of Section II, it is only necessary to choose a suitable Viterbi decoder performance function  $f(x)$ . The function  $f(x)$  for  $x$  larger than the node synchronization threshold is taken to be the ideal performance function as exhibited in Ref. 6. For values of  $x$  below the threshold,  $f(x)$  is assumed to be equal to 1/2, a value which represents a random decoder output.

### IV. Numerical Results

The performance of the concatenated channel with radio losses and node synchronization losses was computed using the mixed-rate model of Section II together with the Viterbi decoder performance function in Section III. Performance curves were generated for carrier loop SNRs of between 10 and 20 dB and for node synchronization thresholds of between 0.0 and 2.5 dB. Some of the results are plotted in Fig. 4. The predicted performance of the concatenated chan-

nel with a node synchronization threshold of 2.0 dB and carrier tracking loop SNRs of 11, 12, and 13.5 dB is shown along with some actual hardware test data generated in the Telecommunication Development Laboratory (TDL) (Ref. 5). The close agreement between the actual data and the predicted performance corroborates the mixed-rate model.

Graphs of Reed-Solomon decoded bit error rate performance for node synchronization thresholds of 0.0 and 2.0 dB appear in Figs. 5 and 6 respectively for various loop SNRs. These were used to generate the radio loss curves shown in Fig. 7. Radio loss is defined to be the additional energy per bit (in dB) needed to achieve some predetermined bit error rate in a system with imperfect carrier tracking compared to a system with ideal carrier tracking. In this case, the ideal system is taken to be one in which the node synchronization threshold of the Viterbi decoder is equal to 0 (or minus infinity dBs). The performance of this ideal system is therefore given by the mixed-rate model with  $f(x)$  equal to the ideal performance function of Ref. 6. The fixed bit error rate is  $10^{-5}$ , the concatenated bit error rate needed for transmission of compressed imaging data.

The difference between the two curves in Fig. 7 represents the incremental SNR loss due to poor node synchronization performance. Data from design control tables predict that for Voyager 2 Uranus encounter at a modulation index of  $76^\circ$  and 90% weather, a 64-meter antenna would have an associated loop SNR of about 13.2 dB. A four-element array consisting of one 64-meter antenna and three 34-meter antennas would have a combined loop SNR of 15.4 dB, provided a combined carrier referencing scheme (Refs. 6, 7) is used. The associated node synchronization losses for the concatenated channel would be 0.89 dB for the 64-meter antenna alone and 0.57 dB for the array. The arrayed loss would be slightly worse if baseband-only combining were used. For Neptune encounter with a  $74^\circ$  modulation index, the loop SNR of the 64-meter antenna would be 10.8 dB while that of the array would be about 13.6 dB. These correspond to node synchronization losses of 1.25 dB and 1.0 dB respectively.

The losses for the concatenated channel due to poor node synchronization performance are plotted as a function of node synchronization threshold in Fig. 8 for various carrier tracking loop SNRs. These represent the additional degradations over the carrier phase tracking losses. The performance points for the 64-meter antenna and the four-element array for Voyager 2 Uranus and Neptune encounters are included in the figure. It is evident from the figure that there is much to be gained by improving the performance of the Viterbi decoder's node synchronization algorithm.

## V. Conclusions

If one compares the node synchronization losses for the concatenated channel (Fig. 8) with those for the convolutional-only channel (Ref. 3), it is clear that the concatenated channel is more sensitive to this type of degradation. The difference becomes more pronounced at higher carrier tracking loop SNRs, becoming about a 0.1-dB difference at a loop SNR of 15 dB and a node synchronization threshold of 2.0 dB. This is not a significant additional loss. The conclusion that must be drawn is that poor node synchronization by itself does not degrade the concatenated channel much more than it does the convolutional-only channel.

There are other related factors, however, that may greatly increase the effects of poor node synchronization on the con-

catenated link. When the Viterbi decoder decides that it is out of synchronization, it might add or delete a channel symbol from the data stream. This could result in a loss of codeword synchronization (or frame synchronization) in the Reed-Solomon decoder. If frame synchronization is lost, it is likely that several Reed-Solomon frames might be lost before resynchronization occurs. This could mean that thousands of bits would be only Viterbi decoded. The model described in this report does not consider the effects of frame synchronization losses.

Other sources of degradation that should be considered are imperfect subcarrier tracking and demodulation and imperfect symbol tracking. These effects are currently under investigation and will be the subject of a subsequent report.

## References

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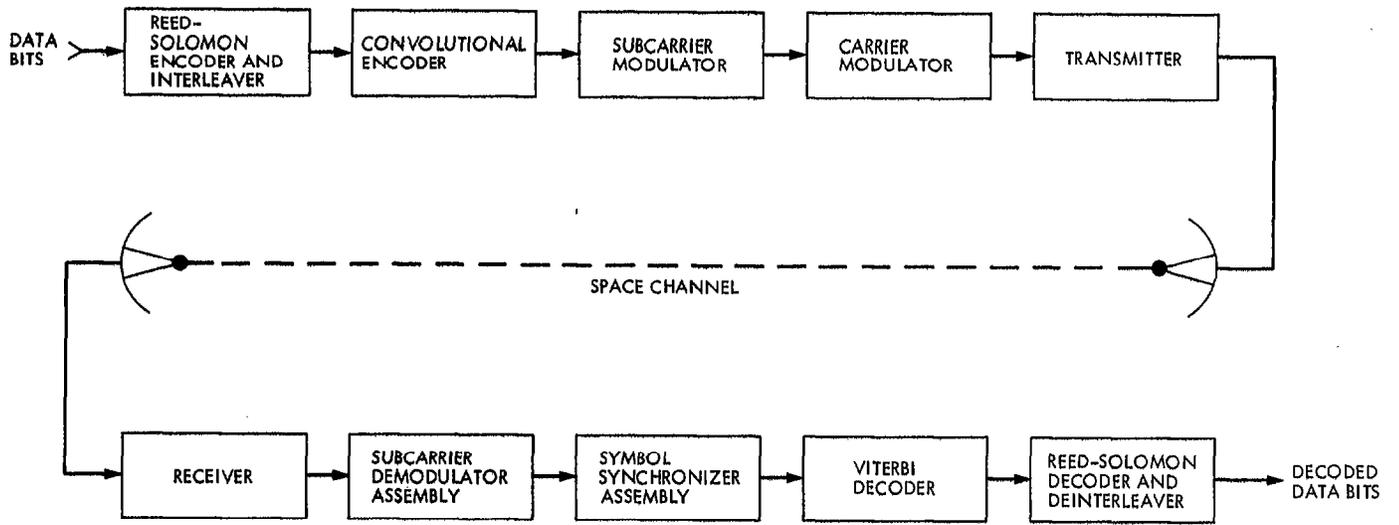


Fig. 1. Concatenated coding system block diagram

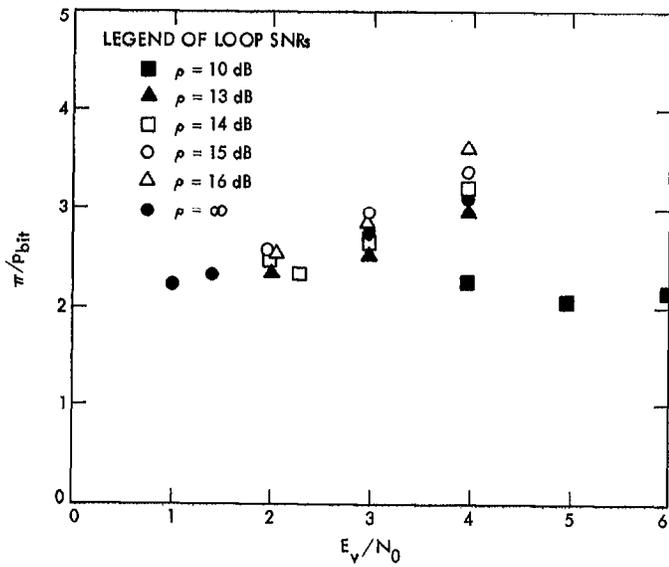


Fig. 2. The relationship between Viterbi decoder bit error rate  $\rho$  and Reed-Solomon symbol error rate  $\pi$

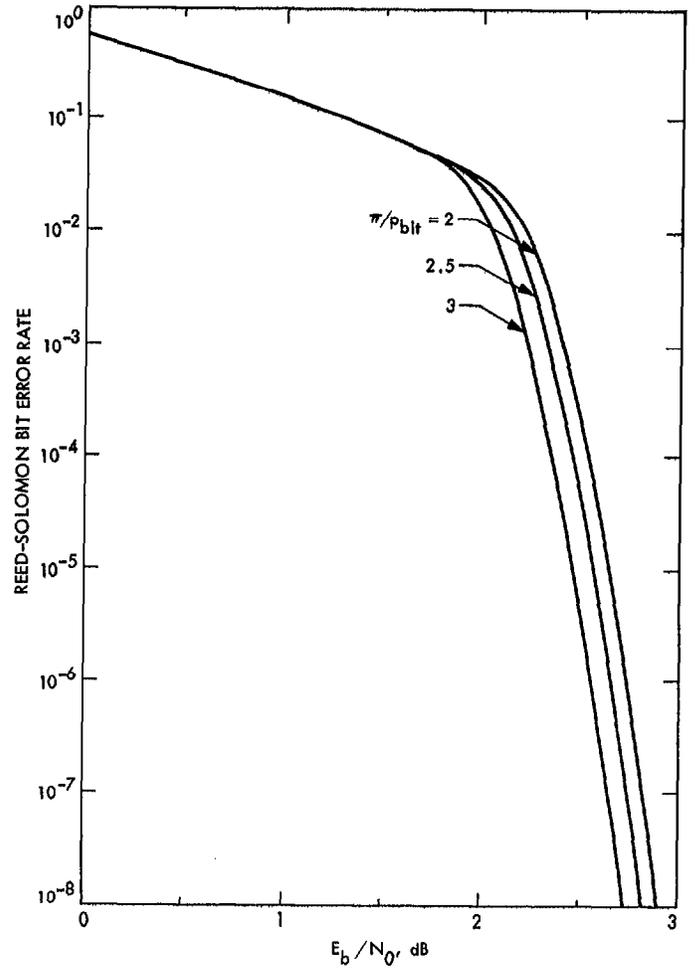


Fig. 3. Concatenated performance assuming that  $\pi/p_{bit}$  is a constant

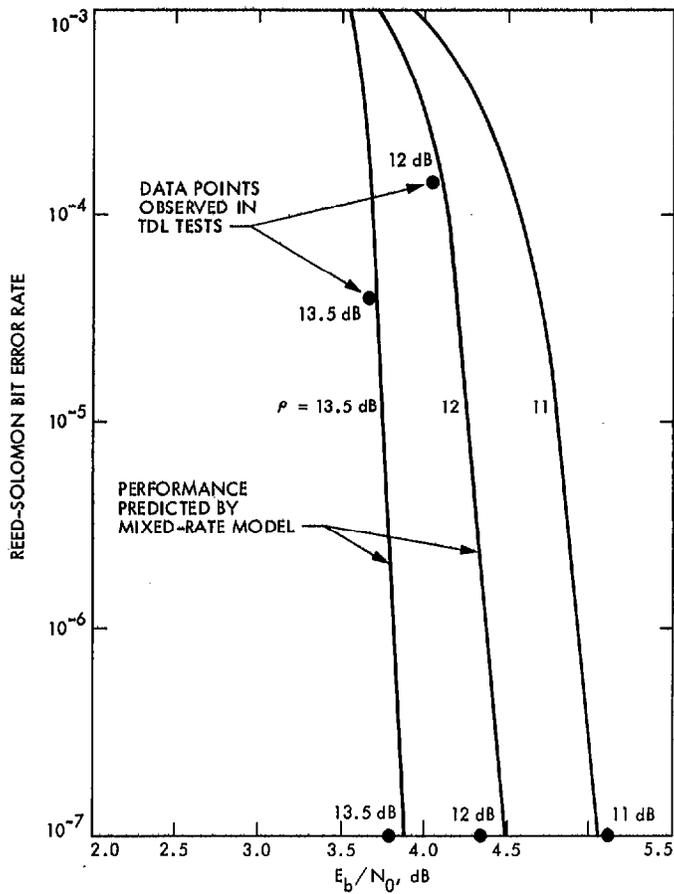


Fig. 4. A comparison of the mixed-rate model with node synchronization losses to hardware test data

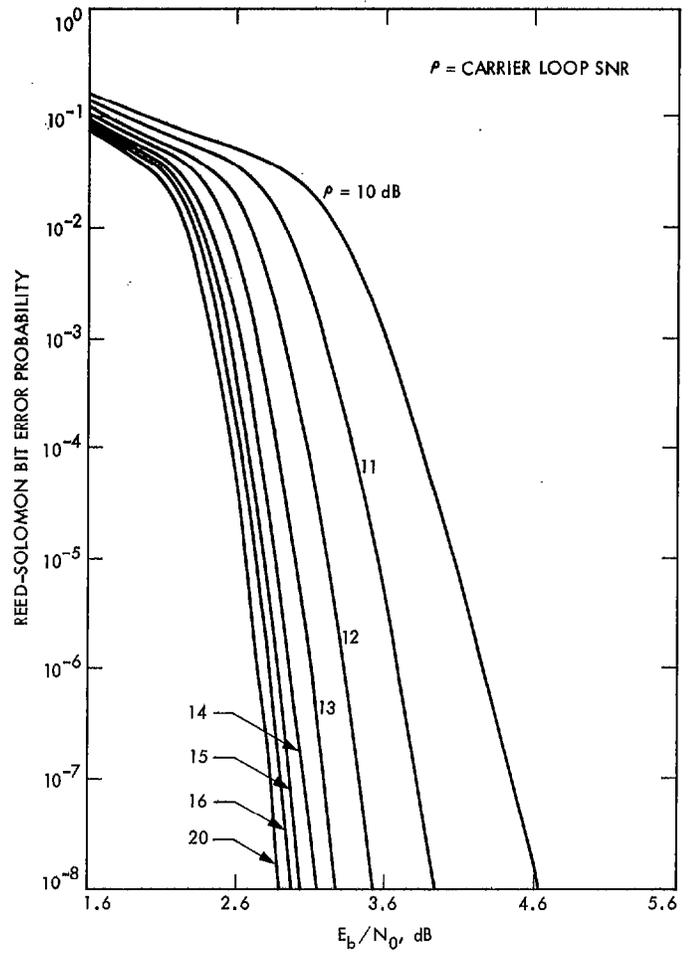
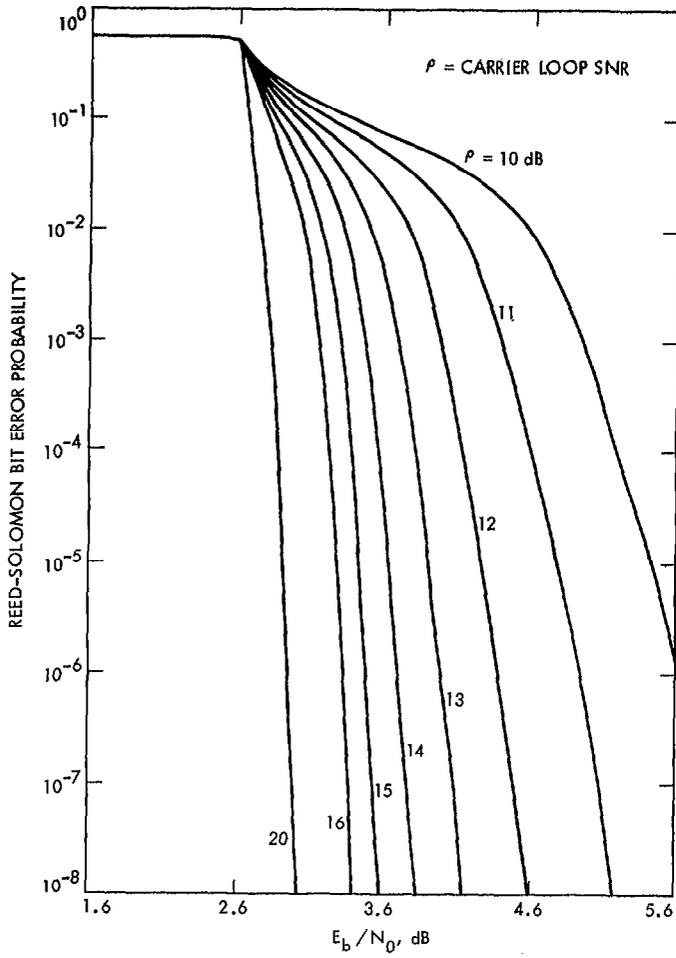
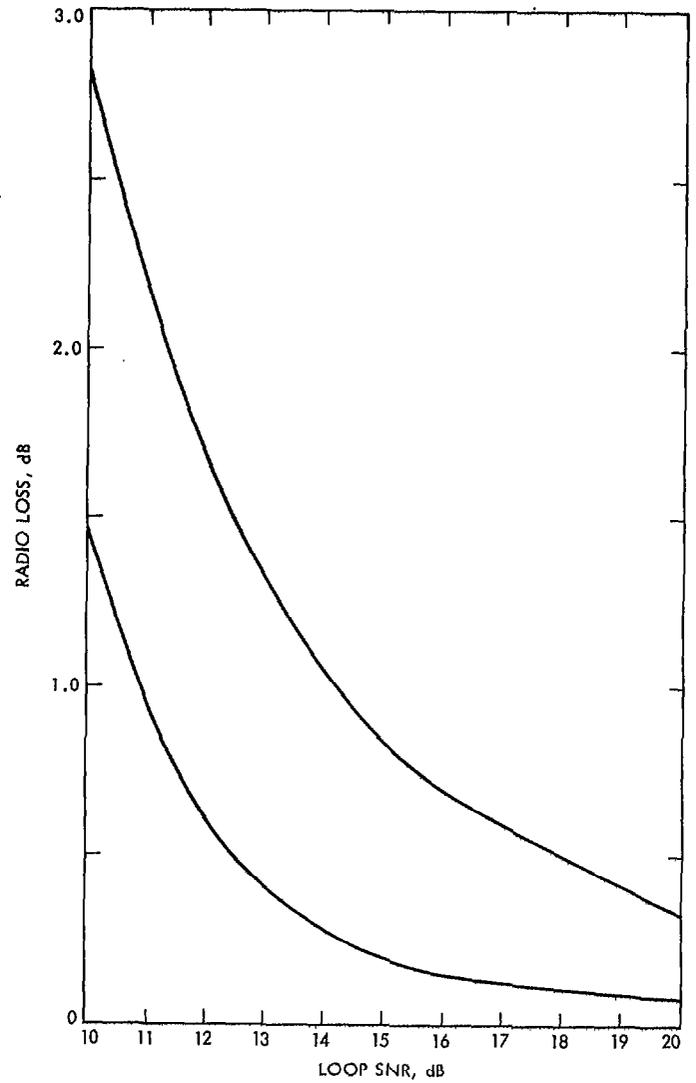


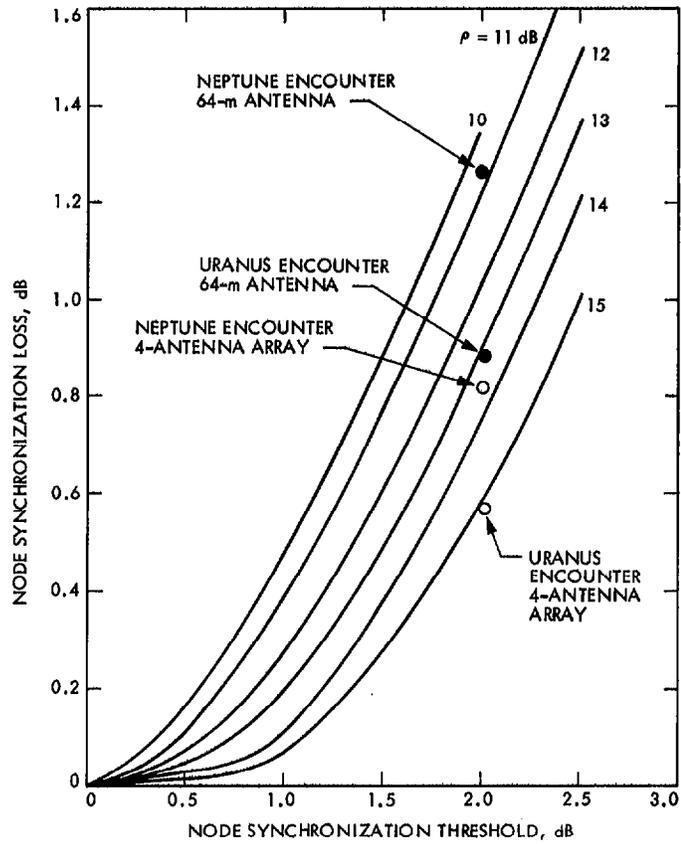
Fig. 5. Performance of the concatenated channel: node synchronization threshold = 0.0 dB



**Fig. 6. Performance of the concatenated channel: node synchronization threshold  $\approx 2.0$  dB**



**Fig. 7. Radio loss for the concatenated channel at  $\text{BER} = 10^{-5}$  and node synchronization thresholds of 0.0 and 2.0 dB**



**Fig. 8. Node synchronization losses for the concatenated channel at various carrier tracking loop SNRs  $\rho$**