

Virtual Center Arraying

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One way to increase the amount of data that can be received from outer planet missions is to array several ground antennas in such a way as to increase the total effective aperture of the receiving system. One such method is virtual center arraying (VCA). In VCA, a combined carrier reference is derived at a point that is, conceptually, the geometric center of the array. This point need not coincide with any of the actual antennas of the array. This report includes a noise analysis of the VCA system and exhibits formulas for the phase jitter as a function of loop bandwidths and the amount of loop damping. If the ratio of the loop bandwidths of the center loop to the vertex loops is greater than 100, then the jitter is very nearly equal to that expected for ideal combined carrier referencing.

I. Introduction

Many different antenna arraying systems have been proposed for increasing the data transmission rates from outer planet missions. Two of these schemes, "baseband only combining" and "baseband combining with combined carrier referencing" (or simply "combined carrier referencing"), have been studied in Ref. 1. In ideal combined carrier referencing, the loop signal-to-noise ratio (loop SNR) of the array is equal to that of a single aperture that is the sum of the effective apertures of the individual array elements. In practice, however, it is not possible to achieve ideal combined carrier referencing. Two conceptually different systems have been proposed to achieve performance approaching that of ideal combined carrier referencing.

The approach proposed in Ref. 2 will be referred to as the "master-slave" system. In the master-slave system (see Fig. 1), the carrier power from each element of the array is combined at one of the antenna receivers, called the "master." This

receiver derives a combined carrier reference that is used to carrier-aid the other receivers, called "slaves." A short loop in each slave receiver is used to track the frequency and phase differences between the master and local signal. A major benefit of carrier aiding is that the bandwidths of these short loops may be made narrower than would otherwise be possible, since the master receiver provides a good estimate of doppler-induced phase drifts. This bandwidth narrowing produces higher loop SNR's in the slave receivers. The performance of the master-slave system has been studied extensively (Refs. 2, 3, 4).

The approach proposed in Ref. 5 and recently rediscovered by J. W. Layland will be referred to as "virtual center arraying" (VCA). In VCA (see Fig. 2), a combined carrier reference is derived at a point called the center. This point need not coincide with any of the actual array elements. The combined carrier reference is used to carrier-aid the individual receivers, called "vertices." Short loops in the vertices track the frequency and phase differences in the carrier reference. The

vertex short loops, like the slave loops from the master-slave system, may be made narrow because of the carrier aiding. The master-slave system requires one less short loop than VCA. Also, both systems perform like ideal combined carrier referencing in the limit as the short loop bandwidths approach zero.

The VCA system has received less attention than the master-slave system. While a study of the stability of Layland's version of VCA appears in Ref. 6, the performance of VCA has not been previously determined. In Section II, this performance is determined by developing an expression for phase jitter as a function of loop bandwidths and the amount of loop damping. In Section III, loop jitter is given explicitly for expected Voyager 2 Uranus and Neptune encounter conditions for an array of three 34-m antennas with and without a 64-m antenna. These plots demonstrate that when the ratio of the loop bandwidth of the center loop to those in the vertex short loops is greater than 100, the jitter is very nearly equal to that expected for ideal combined carrier referencing.

II. Jitter Performance of VCA

All signal names in this section correspond to the labeling of Fig. 2. Consider a set of input signals of the form

$$S_k(t) = \sqrt{2} A_k \sin[\omega_0 t + \theta_k(t)] + n_k(t) \quad (k = 1, 2, 3, \dots, N)$$

where A_k is the carrier amplitude, ω_0 is an intermediate frequency, and θ_k is the phase in the k^{th} vertex. The data part of the input signal is assumed to have been eliminated by filtering. The noises n_k are assumed to be independent white Gaussian processes. $C(t)$ is of the form

$$C(t) = \sqrt{2} K_0 \cos[\omega_1 t + \hat{\theta}_0(t)]$$

where K_0 is the center VCO rms output and $\hat{\theta}_0$ is the estimate of the carrier phase in the center. After the first mixer and effective low-pass filtering, the resulting signal is

$$L_k(t) = K_0 A_k \sin[(\omega_0 - \omega_1)t + \theta_k(t) - \hat{\theta}_0(t)] + C(t)n_k(t) .$$

The function W_k is defined by

$$W_k(t) = 2K \cos[(\omega_0 - \omega_1)t + \hat{\theta}_k(t)]$$

where K is the VCO rms output in the vertex loops (assumed to be the same for all the short loops) and $\hat{\theta}_k$ is the phase

estimate in the k^{th} vertex short loop. After the second mixer (and effective low-pass filtering),

$$X_k(t) = K K_0 A_k \sin \phi_k(t) + N_k(t)$$

where

$$\phi_k(t) = \theta_k(t) - \hat{\theta}_k(t) - \hat{\theta}_0(t)$$

and $N_k(t)$ ($k = 1, 2, 3, \dots, N$) are independent narrowband Gaussian processes. Each $N_k(t)$ is assumed to have one-sided spectral density $N_0/2$ in the bandwidth of the short loops.

If $f_k(t)$ is the impulse response of the vertex filter $F_k(s)$, then

$$Y_k(t) = \int_{-\infty}^{\infty} X_k(t-u) f_k(u) du$$

and, if each vertex has VCO gain equal to K_{VCO} ,

$$\begin{aligned} \frac{d\hat{\theta}_k}{dt} &= K_{VCO} Y_k(t) \\ &\approx K_{VCO} K K_0 \int_{-\infty}^{\infty} [A_k \phi_k(t-u) + N_k(t-u)] f_k(u) du \end{aligned}$$

where the last expression holds for small phase errors ϕ_k . After taking Laplace transforms and solving for $\hat{\theta}_k$,

$$\hat{\theta}_k = H_k(s) \left(\theta_k - \theta_0 + \frac{N_k}{A_k} \right)$$

where

$$H_k(s) = \frac{K_T A_k F_k(s)}{s + K_T A_k F_k(s)}$$

is the closed loop transfer function for the k^{th} vertex short loop and

$$K_T = K_{VCO} K K_0 .$$

The summing junction combines the X_k 's with coefficients β_k . These coefficients are normalized so that

$$\sum_{k=1}^N \beta_k^2 = 1 .$$

It is known (see Ref. 1) that the optimal selection for β_k is

$$\beta_k = \frac{A_k}{A_T}$$

where

$$A_T^2 = \sum_{j=1}^N A_j^2.$$

The output of the summing junction is

$$Z(t) = \sum_{j=1}^N \beta_j X_j(t)$$

whence

$$\begin{aligned} \frac{d\hat{\theta}_0}{dt} &= K_{VCO_0} Y_0(t) \\ &= K_{VCO_0} \int_{-\infty}^{\infty} Z(t-u) f_0(u) du \end{aligned}$$

where $f_0(u)$ is the impulse response of the center filter $F_0(s)$.
If

$$K_{T_0} = K_{VCO_0} K K_0$$

then, for small phase error ϕ_j ,

$$\frac{d\hat{\theta}_0}{dt} \approx K_{T_0} \sum_{j=1}^N \beta_j \int_{-\infty}^{\infty} e(u) \{A_j \phi_j(t-u) + N_j(t-u)\} du.$$

It may then be shown that if $H_j = H_1$ for $j = 2, 3, 4, \dots, N$ (i.e., if all the short closed loop bandwidths are equal), then

$$\hat{\theta}_0 = \frac{1}{A_T} \frac{H_0(s)[1-H_0(s)]}{1-H_0(s)H_1(s)} \sum_{j=1}^N \beta_k A_k \left(\theta_j + \frac{N_j}{A_j} \right)$$

where

$$H_0(s) = \frac{K_{T_0} A_T F_0(s)}{s + K_{T_0} A_T F_0(s)}.$$

If each θ_j is assumed to be slowly varying so that $E(\theta_j) \approx \theta_j$ then

$$\begin{aligned} \phi_k - E(\phi_k) &= [1 - H_1(s)] [E(\hat{\theta}_0) - \hat{\theta}_0] - H_1(s) \frac{N_k}{A_k} \\ &= - \frac{1}{A_T} \frac{H_0(s) [1 - H_1(s)]^2}{1 - H_0(s)H_1(s)} \sum_{j=1}^N \beta_k N_k - H_1(s) \frac{N_k}{A_k} \end{aligned}$$

and so the jitter in the k^{th} vertex is given by

$$\begin{aligned} \sigma_{\phi_k}^2 &= \frac{N_0(1-\beta_k^2)}{2A_T^2} \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} |a_1(s)|^2 ds \\ &\quad + \frac{N_0}{2A_T^2} \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \left| \beta_k a_1(s) + \frac{1}{\beta_k} H_1(s) \right|^2 ds \end{aligned}$$

where

$$a_1(s) = \frac{H_0(s)[1-H_1(s)]^2}{1-H_0(s)H_1(s)} = a_2(s) - H_1(s)$$

and

$$a_2(s) = \frac{H_0(s)[1-H_1(s)] + H_1(s)[1-H_0(s)]}{1-H_0(s)H_1(s)}.$$

This may be simplified as follows:

$$\begin{aligned} \sigma_{\phi_k}^2 &= \frac{N_0}{2A_T^2} \left[\frac{1}{2\pi i} \int_{-i\infty}^{i\infty} |a_2(s)|^2 ds \right. \\ &\quad \left. + \left(\frac{1-\beta_k^2}{\beta_k^2} \right) \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} |H_1(s)|^2 ds \right] \end{aligned}$$

If all the filters are taken to be of the second-order having the same damping factor r , then, using integration techniques described in Ref. 7,

$$\sigma_{\phi_k}^2 = \frac{N_0 B_L}{A_T^2} \left[\frac{R^2 + \frac{2r}{1+r}R + 1}{R^2 + R} + \left(\frac{1-\beta_k^2}{\beta_k^2} \right) \frac{1}{R} \right]$$

where B_L is the bandwidth in the center loop and R is the ratio of B_L to the bandwidths in the vertex short loops.

III. Conclusions

It was shown in Ref. 1 that the loop SNR for ideal combined carrier referencing in the presence of bandpass hardlimiting is given by

$$\rho_{CCR} = \frac{A_T^2}{N_0 B_{L1} \Gamma}$$

where B_{L1} is B_L expanded by the effects of the bandpass hardlimiter and Γ is a suppression factor associated with that limiter. Also, the loop jitter for ideal combined carrier referencing is approximately

$$\sigma_{CCR}^2 = \frac{1}{\rho_{CCR}} = \frac{N_0 B_{L1} \Gamma}{A_T^2}$$

Thus, if bandpass hardlimiting is added to VCA before the first mixers, then the expression for the jitter in the k^{th} vertex becomes

$$\sigma_{\phi_k}^2 = \sigma_{CCR}^2 \left[\frac{R^2 + \frac{2r}{1+r}R + 1}{R^2 + R} + \left(\frac{1 - \beta_k^2}{\beta_k^2} \right) \frac{1}{R} \right]$$

The loop SNR of VCA in the k^{th} vertex is then given by

$$\rho_k = \rho_{CCR} \left[\frac{R^2 + \frac{2r}{1+r}R + 1}{R^2 + R} + \left(\frac{1 - \beta_k^2}{\beta_k^2} \right) \frac{1}{R} \right]^{-1}$$

Notice that

$$\lim_{R \rightarrow \infty} \rho_k = \rho_{CCR}$$

Graphs of ρ_k plotted as a function of R are exhibited in Fig. 3. Arrays consisting of three 34-m antennas with and without a 64-m antenna are considered under typical Voyager 2 Uranus and Neptune encounter conditions.

References

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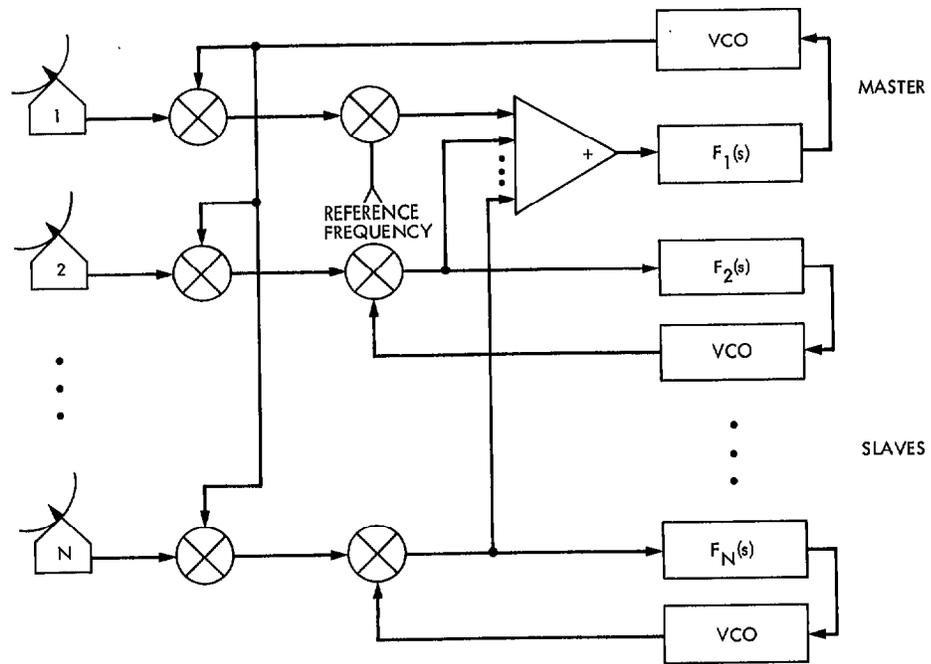


Fig. 1. Block diagram of master/slave scheme

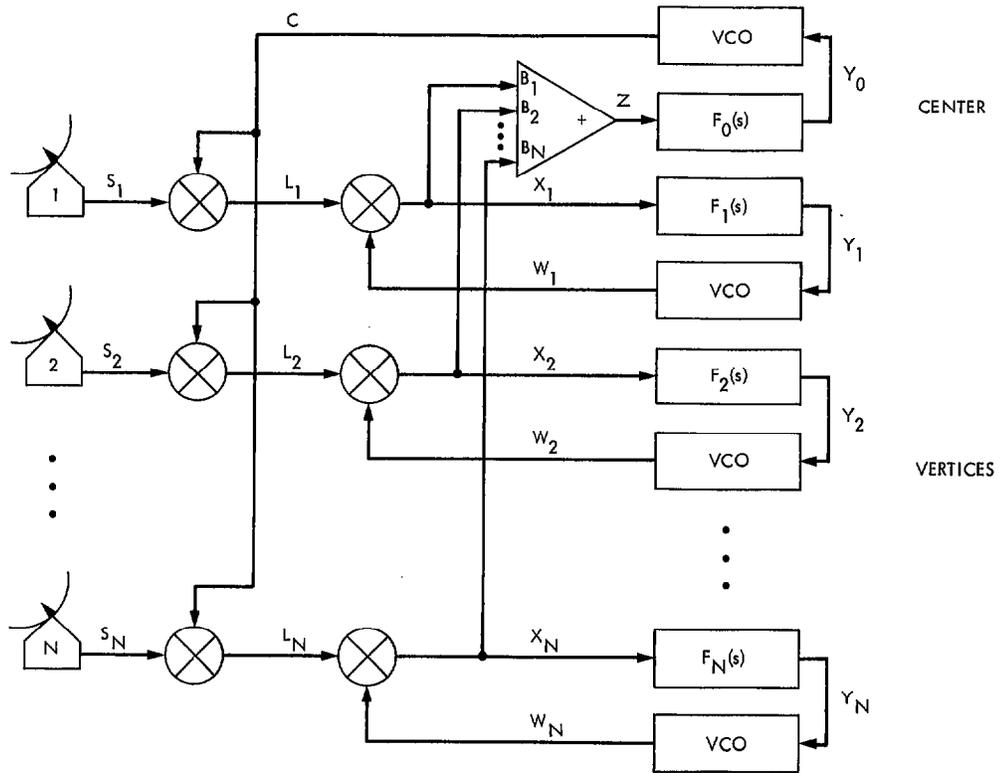


Fig. 2. Block diagram of virtual center arraying scheme

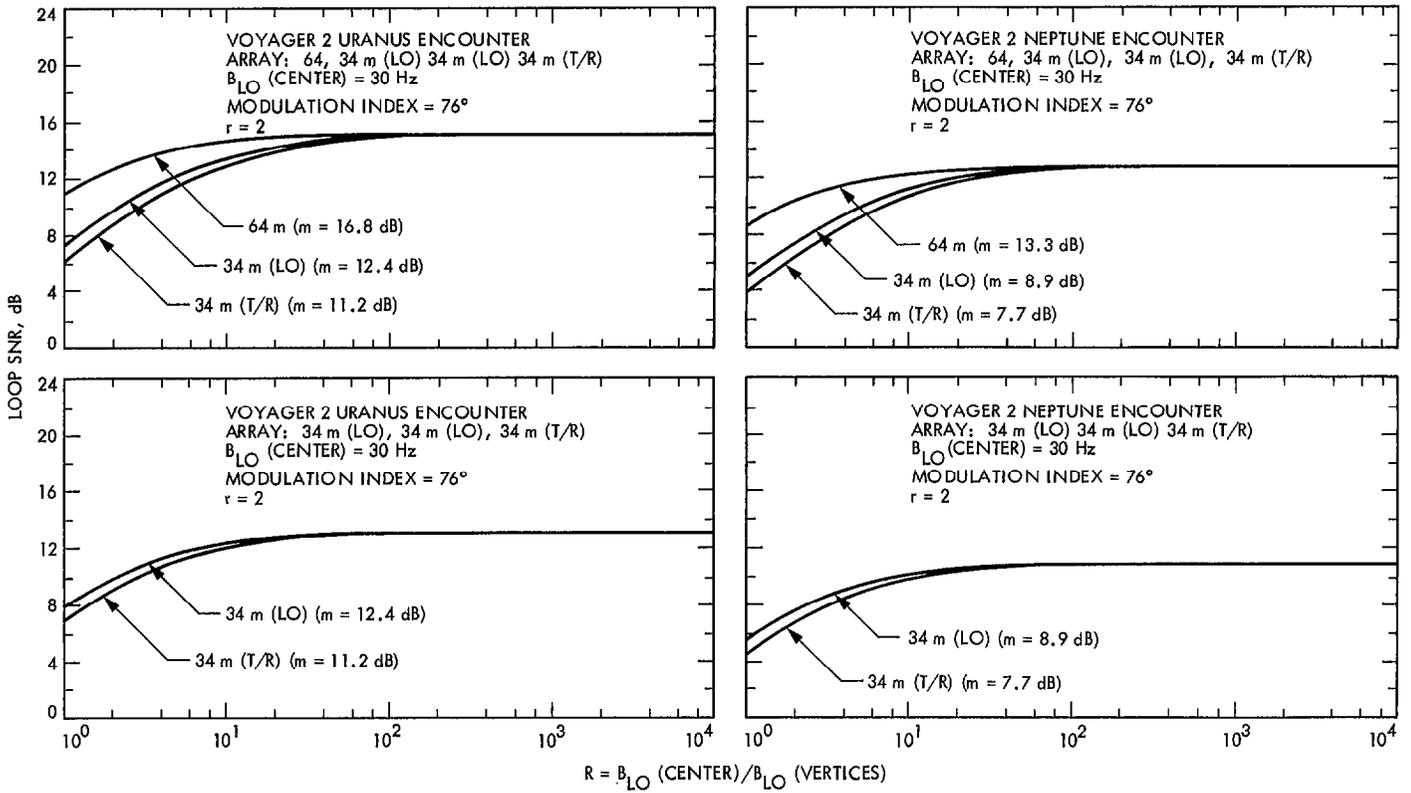


Fig. 3. Virtual center arraying performance