Tropospheric Path Length Fluctuation in Temperate Semiarid Locales: Application to the Gravitational Wave Detection Experiment

A. L. Berman
TDA Mission Support Office

S. D. Slobin
Radio Frequency and Microwave Subsystems Section

Wet tropospheric path length fluctuation will, at some level of system sensitivity, begin to interfere with the search for gravitational waves using the spacecraft doppler method. This article investigates radiosonde data from Edwards Air Force Base and radio metric scintillation data collected over a long, nearly horizontal path in Hawaii. Utilizing a previous hypothesis that wet tropospheric path length fluctuation is proportional to total wet tropospheric signal delay, the two types of data are shown to be in reasonable agreement for averagings times (τ_a) of approximately 3000 seconds. The two-way modeled tropospheric fractional frequency fluctuation at τ_a = 1000 seconds is 1.6 $\times$ 10^{-14}.

I. Introduction

Proposals have recently been advanced to search for gravitational waves in ultraprecise two-way doppler data. Preliminary estimates of gravitational wave characteristics indicate that a total measurement system fractional frequency fluctuation ($\sigma(\Delta F/F)$, where F is an S- or X-band frequency) of 1 $\times$ 10^{-15} over the time scales of interest (50 to 5000 seconds) will be required (Ref. 1). At this level, fluctuations in the wet (water vapor) component of the tropospheric signal delay (R_w) will surely constitute a major error source.

In a previous article (Ref. 2), Berman modeled wet tropospheric fluctuation under the *assumption* that wet tropospheric path length fluctuation is proportional to the total wet tropospheric signal delay. In this article, a very low frequency wet tropospheric path length fluctuation spectrum is constructed from radiosonde data measured at Edwards Air Force Base, and a high frequency fluctuation spectrum is constructed from tropospheric radio metric scintillation data taken by Thompson (Ref. 3) in Hawaii. It is shown that the two mean spectra are reasonably consistent for fluctuation frequencies (ν) $\sim$ 3 $\times$ 10^{-4} Hz under the assumption that the wet tropospheric path length fluctuation is proportional to the total wet tropospheric delay.

II. Radiosonde Data

Wet zenith tropospheric signal delay values (R_{wz}, cm) were computed from radiosonde measurements made at Edwards AFB during June through November 1977 and April 1978 through March 1979. Edwards AFB is located in the Mojave Desert about fifty miles north of Los Angeles and has a climate similar to that of the Deep Space Network (DSN) Goldstone tracking complex. The radiosonde measurements were made at irregular intervals ranging from periods of several hours to approximately one day. Changes in zenith delay (ΔR_{wz}) as a function of separation time (τ_a) were computed for all measurements within 3 days of each other. These were then sorted into "bins" of 1/10-day each and the mean value of each bin calculated. Figures 1 and 2 display these mean bin averages (in cm).

As previously noted, Berman (Ref. 2) has hypothesized that *fluctuation* in wet tropospheric delay is proportional to the *total* wet tropospheric delay. To test this hypothesis, the changes in wet zenith delay (ΔR_{wz}) were correlated with mean zenith delay (R_{wz}) for each of the bins in both data sets. Computed correlation was significant in almost every case, with the average computed correlation coefficient (r) being:

1977 data (6 months):	$r = 0.59$
1978-1979 data (12 months):	$r = 0.26$

Based on this observed correlation, it is considered that the heuristic assumption of proportionality between wet tropospheric fluctuation and mean wet tropospheric delay continues valid.

To utilize this relationship, each of the changes in wet tropospheric delay was normalized ($\Delta R_{wz}/R_{wz}$) by the average value of the delay. Again, these were sorted into 1/10-day (2.4-hour) bins. Figures 3 and 4 present the mean bin values for the normalized (fractional) fluctuations. The lines in Figs. 3 and 4 represent least squares linear curve fits to these data. The equivalent parametric forms for these data fits are (τ_a in seconds):

1977 data (6 months):

$$\sigma_{R_{wz}}(\tau_a) = 0.079 R_{wz} \left(\frac{\tau_a}{1000}\right)^{0.24}$$

1978-1979 data (12 months):

$$\sigma_{R_{wz}}(\tau_a) = 0.069 R_{wz} \left(\frac{\tau_a}{1000}\right)^{0.33}$$

These can be compared to the estimate Ref. 2 made based on a very preliminary examination of tropospheric fluctuation obtained from water vapor radiometer data:

$$\sigma_{R_{wz}}(\tau_a) = 0.02 R_{wz} \left(\frac{\tau_a}{1000}\right)^{0.6}$$

Whereas the radiosonde data show greater $\tau_a = 1000$ second (17 minute) fluctuation that the water vapor radiometer data (7% versus 2%), the one-day modeled fluctuations are comparable at 27% and 29%, respectively.

In the remainder of the article, the 1977 and 1978–1979 data fits will be combined into a single model as follows:

$$\sigma_{R_{wz}}(\tau_a) = 0.074 R_{wz} \left(\frac{\tau_a}{1000}\right)^{0.29}$$

A fractional frequency fluctuation may be defined for this data:

$$\sigma(\Delta F/F) = \sigma_{R_{wz}}(\tau_a)/c\tau_a$$

where

c = velocity of light, cm/second

τ_a = doppler averaging time, seconds[1]

The fractional frequency fluctuation for the Edwards AFB radiosonde data thus becomes

$$\sigma(\Delta F/F) = 2.0 \times 10^{-14} \left(\frac{\tau_a}{1000}\right)^{-0.71}$$

when a (yearly average) value of 8 cm is assumed for R_{wz}.

III. Tropospheric Radio Metric Scintillation Data

In Ref. 3, Thompson, et al., describes the phase spectral density of wet tropospheric signal delays over a 64-km Hawaiian (nearly horizontal) range. The fluctuation frequency (ν) range of validity for this data is

$$3 \times 10^{-3}\ \text{Hz} < \nu < 3 \times 10^{-1}\ \text{Hz}$$

To compare these data to the Edwards AFB data, one must scale the results by the appropriate total wet delays. The exact tropospheric water vapor distribution at the time of the Thompson experiment over the 64-km Hawaii radiometer

[1]For the remainder of this article, "measurement separation time" will be equated with "doppler averaging time."

range is unknown. Using the U.S. Standard Tropical Atmosphere, 15° N (Ref. 4), it is estimated that there were a total of 87 gm/cm^2 precipitable water along the 64-km path. A standard year-average zenith atmosphere at Edwards AFB has 1.3 gm/cm^2 of precipitable water. This ratio of nominal delays (proportional to integrated water vapor) is 66.9 and therefore the phase spectral density of the Thompson data must be scaled downward by $(66.9)^2$. Thus, the Edwards AFB equivalent zenith troposphere X-band phase spectral density (based on Hawaii measurements) is

$$P_\phi(\nu) = 1.09 \times 10^{-7}\, \nu^{-2.57} \text{ rad}^2 \text{ Hz}^{-1}$$

$$3 \times 10^{-3} \text{ Hz} < \nu < 3 \times 10^{-1} \text{ Hz}$$

Reference 5 provides the following expression (with the Ref. 6 correction) for Allan variance ($\sigma_y{}^2$) derived from phase spectral density (ν_0 is transmission frequency):

$$\sigma_y^2(\tau_a) = 4\pi^{-2}\nu_0{}^{-2} A\tau_a{}^{a-3} \left[\int_0^\infty Z^{-a} \sin^4(\pi Z)\, dZ\right]$$

where A and a are defined from the expression for phase spectral density $P_\phi(\nu) = A\nu^{-a}$ (as previously given).

Hence from the above one obtains from the Thompson data as corrected to Edwards AFB, for X-band frequency (ν_0 = 8.4 GHz) and a nominal 8-cm delay:

$$\sigma_y(\tau_a) = 1.15 \times 10^{-14} \left(\frac{\tau_a}{1000}\right)^{-0.22}$$

The radiosonde fractional frequency fluctuation and radio metric scintillation Allan variance data[2] are plotted in Fig. 5. As can be seen, they are reasonably consistent. In fact, the steeper slope of the very low frequency (Edwards AFB) data is reasonable, since one would expect the tropospheric fluctuation spectrum to begin to flatten at about 3 days (4×10^{-6} Hz) due to movement of large-scale air masses.

[2]For the purpose of this article, the radiosonde fractional frequency fluctuation is considered to be equal to the square root of the Allan variance.

IV. Comparison of Radiosonde and Radio Metric Scintillation Phase Spectral Density

By equating the radiosonde fractional frequency fluctuation (squared) with the previous expression for Allan variance, a value of A may be deduced. Using this procedure, a phase spectral density for Edwards AFB radiosonde data is obtained:

$$P_\phi(\nu) = 6.76 \times 10^{-14} \nu^{-1.58} \text{ rad}^2 \text{ Hz}^{-1}$$

over the range of validity $3 \times 10^{-6} < \nu < 10^{-4}$.

Figure 6 shows the phase power spectra for both the Hawaii microwave data and the radiosonde data taken at Edwards AFB. The microwave data have been normalized to year-average zenith values for Edwards AFB; thus, both curves represent phase spectra for the troposphere in temperate semi-arid regions. As stated in Section III, the spectrum at very low frequencies flattens out due to the decrease in long-term troposphere variation for periods greater than a few days. Therefore, the "combination" of the model segments as in Figs. 5 and 6 is intuitively agreeable.

V. Tropospheric Model

Following Ref. 2, a factor of $(\sin\theta)^{-1}$ is added to account for non-zenith delays, where θ = elevation angle. One then has a "combined" radiosonde/radio metric scintillation model for tropospheric fluctuation (cm):

$$\sigma_{R_w}(\theta, R_{wz}, \tau_a) = 0.074\,(\sin\theta)^{-1} R_{wz} \left(\frac{\tau_a}{1000}\right)^{0.29},$$

$$\tau_a > 3000 \text{ s}$$

$$\sigma_{R_w}(\theta, R_{wz}, \tau_a) = 0.043\,(\sin\theta)^{-1} R_{wz} \left(\frac{\tau_a}{1000}\right)^{0.79},$$

$$\tau_a < 3000 \text{ s}$$

Edwards AFB is a reasonable representative of a temperate, semiarid climate, as are the Deep Space Network stations. Table 1 gives the seasonal variation of the mean wet zenith tropospheric signal delay during 1977 through 1979. From this data, a mean yearly value of 8 cm was computed, with a mean summer extreme of 16 cm and a mean winter extreme of

4 cm. The combined radiosonde/radio metric scintillation model for these conditions is presented in Fig. 7.

VI. Discussion

Previous work (Ref. 2) modeled wet tropospheric path length fluctuation under the heuristic hypothesis of proportionality between wet tropospheric path length fluctuation and total wet tropospheric signal delay. In this article, radiosonde measurements collected over 1.5 years at Edwards AFB are used to demonstrate the soundness of this hypothesis. Utilizing the hypothesis, wet zenith tropospheric fractional frequency fluctuation computed from the Edwards AFB radiosonde data and the Hawaiian radio metric scintillation Allan variance data are shown to be reasonably consistent in the region of $\tau_a \sim 3000$ seconds. The fractional frequency fluctuation at τ_a = 1000 seconds is approximately 1.1×10^{-14}, or about a factor of two higher than the value estimated in Ref. 2 from water vapor radiometer data. Since this is a "one-way" measurement, a factor of $\sqrt{2}$ is applied to obtain the equivalent "two-way" value of 1.6×10^{-14}. Since a value this large will certainly impact any attempts to search for gravitational waves using the spacecraft doppler method, there exists a clear need to measure wet tropospheric path length fluctuation at τ_a = 1000 seconds, instead of relying on extrapolations into this critical fluctuation region. The water vapor radiometer is suggested as a possible instrument for measuring tropospheric path fluctuation at τ_a = 1000 seconds, and perhaps ultimately, providing wet tropospheric calibration as well.

References

1. Berman, A. L., "The Gravitational Wave Detection Experiment: Description and Anticipated Requirements," in *The Deep Space Network Progress Report 42-46*, pp. 100-108, Jet Propulsion Laboratory, Pasadena, Calif., Aug. 15, 1978.
2. Berman, A. L., "Parametric Modeling of Low-Frequency Water-Vapor-Induced Tropospheric Path Length Fluctuations" in *The Deep Space Network Progress Report 42-47*, pp. 72-76, Jet Propulsion Laboratory, Pasadena, Calif., Oct. 15, 1978.
3. Thompson, M. C., et al., "Phase and Amplitude Scintillations in the 10 to 40 GHz Band," *IEEE Transactions on Antennas and Propagation, Vol. AP-23, No. 6*, Nov. 1975.
4. Valley, S. L., Editor, *Handbook of Geophysics and Space Environments*, 1965 Edition, p. 2-10, McGraw Hill Book Company, Inc., New York, 1965.
5. Armstrong, J. W., Woo, R., and Estabrook, F. B., "Interplanetary Phase Scintillation and the Search for Very Low Frequency Gravitational Radiation," in *The Astrophysical Journal, Volume 230*, June 1, 1979.
6. Armstrong, J. W., Estabrook, F. B., and Woo, R. "Corrections to Allan Variances...," *Astrophysical Journal* (submitted).

Table 1. Mean zenith wet tropospheric signal delay (R_{wz}) as a function of season during 1977–1979

Period	Delay, cm
January–March	6
April–June	8
July–September	11
October–December	7
Yearly Average	8
Mean Summer Extreme	16
Mean Winter Extreme	4

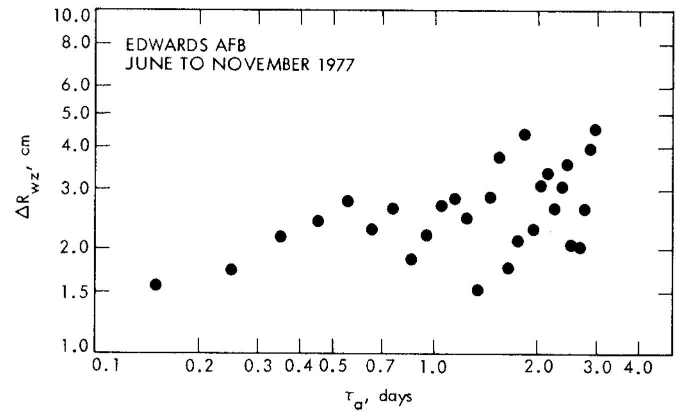

Fig. 1. Zenith wet tropospheric path length change versus averaging time, 1977

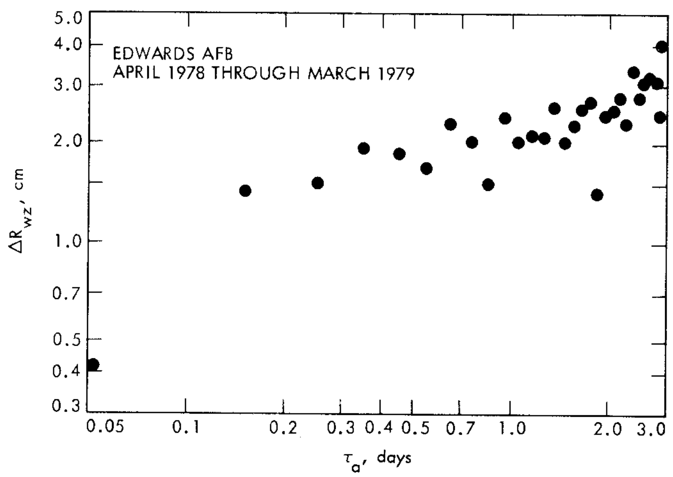

Fig. 2. Zenith wet tropospheric path length change versus averaging time, 1978–1979

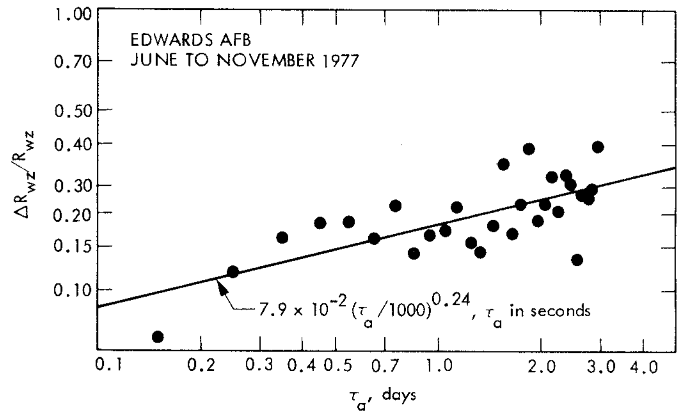

Fig. 3. Zenith wet tropospheric fractional path length change versus averaging time, 1977

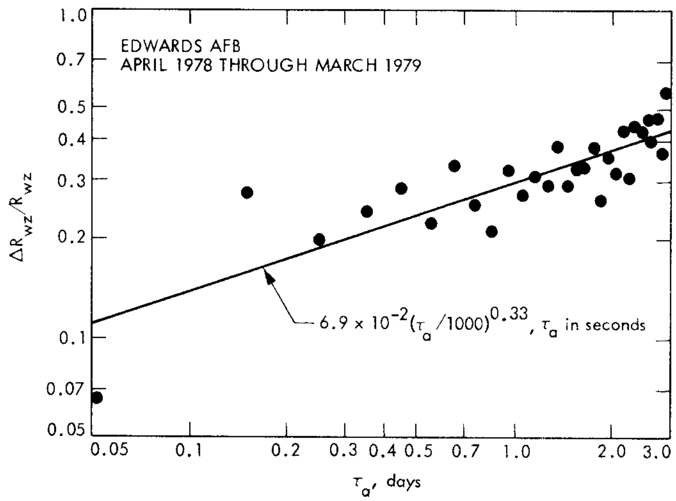

Fig. 4. Zenith wet tropospheric fractional path length change versus averaging time, 1978–1979

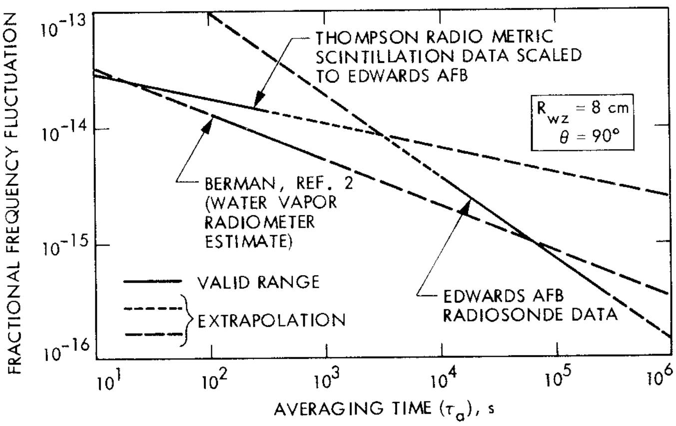

Fig. 5. Zenith wet tropospheric fractional frequency fluctuation versus averaging time

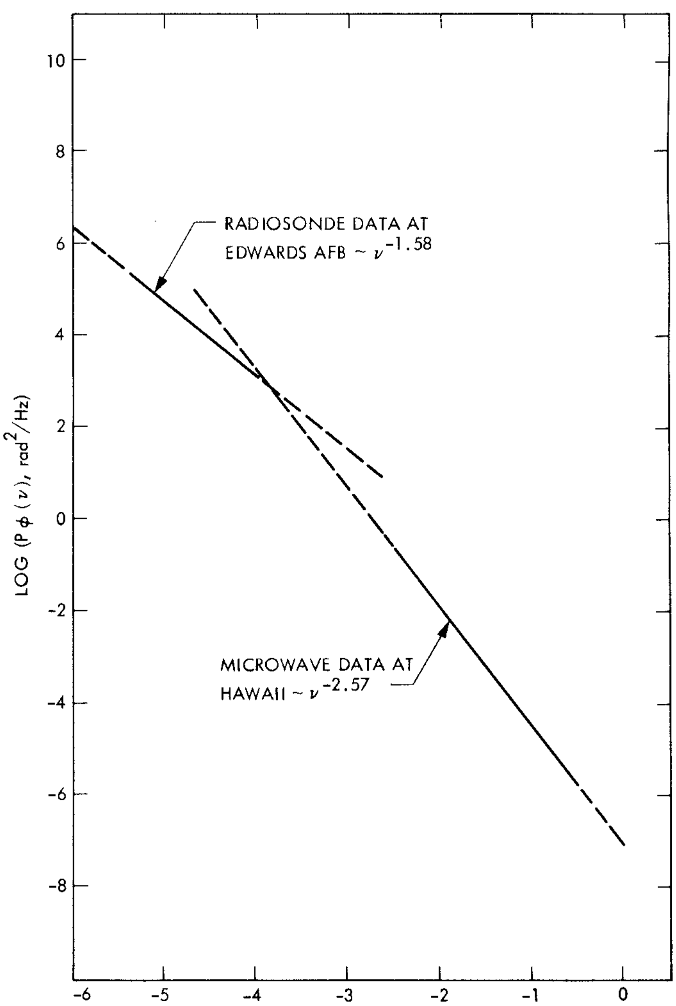

Fig. 6. Phase spectral density computed from radiosonde and radio metric data

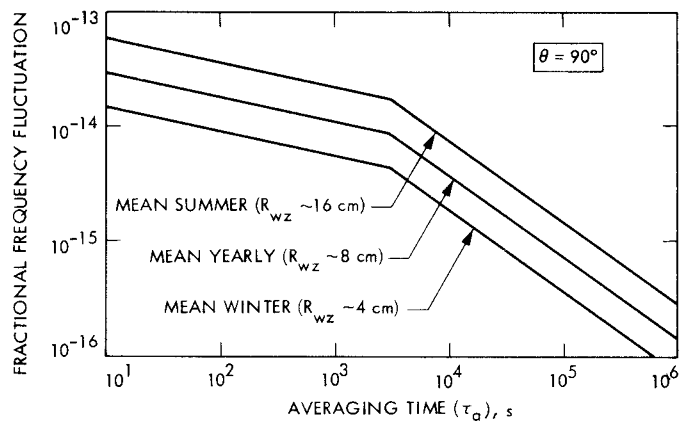

Fig. 7. Seasonal variations in combined radiosonde/radio metric scintillation model