A New, Nearly Free, Clock Synchronization Technique

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A new, near real-time, method for intercomplex clock synchronization is proposed. The method consists of transmitting a symmetric frequency ramp to a spacecraft and determining the time at which the received ramp (in doppler residuals) occurs at overlapping stations. Adjusted preliminary data suggest that the accuracy of the method may be better than 0.7 microseconds. The method requires no additional hardware and can be done during normal tracks. Other, perhaps more accurate, variations of the method are under investigation.

I. Introduction

Interstation clock synchronization is extremely important in both spacecraft navigation and in the ultimate accuracy of scientific data obtained from the spacecraft. Several methods of clock synchronization are either in use or have been proposed. Most precise methods utilize radio signals from extraterrestrial objects – either originating there (Very Long Baseline Interferometry) or bounced from them (Moon Bounce). There are several contributors to the ultimate accuracy of the synchronization which all such methods have in common. These are: 1) station location uncertainty, 2) atmospheric delays, and 3) electronic (station) delays. The new method proposed below is no exception. These three factors, however, can be measured to an accuracy which translates to less than 10 ns synchronization uncertainty.

Currently the DSN synchronizes its clocks via the "Moon Bounce" method. The ultimate accuracy of this method is approximately 5 microseconds. This system requires additional antennas and related hardware, an operator at each site, the generation of special "Moon Bounce" predicts, and the data must be collected and reduced. The method proposed below can accomplish clock synchronization to a similar (and even better) accuracy, using existing data outputs – but it does not require special antennas, predicts, or additional operators. Also, real-time programs can be written to compute certain parameters which will reduce analysis time and give ultimately near-real-time synchronization. A variation of the method can also be utilized for Very Long Baseline Interferometry (VLBI) clock synchronization. This will be discussed later.

II. Description of Method

The method consists of transmitting a symmetric sawtooth frequency ramp to a spacecraft. This ramp can be identical to those currently employed for Pioneer ramp ranging. The transmit time should be such that the ramp is received approximately in the middle of the overlap interval of two DSS whose clock off-sets are to be determined. Both DSS must be locked to the signal throughout the duration of the ramp.

The ramp, unmodeled in predicts, will appear inverted in the doppler residuals. This will be the data base for the synchronization. Doppler residuals are computed by the Real

Time Monitor (RTM) by differencing actual received (DSN) doppler and predicted doppler. For most tracks this difference will be small – a few Hz. For the purpose of this synchronization, the actual magnitude is unimportant. However, the residuals may vary in a nonlinear manner. This is likely when the spacecraft experiences non-constant accelerations. These orbit modeling uncertainties will produce large nonlinear doppler residuals when the doppler changes rapidly. Choosing spacecraft which undergo constant acceleration during the ramp duration will eliminate this problem. Also, this effect can easily be seen in the data so that the data can be corrected or rejected. The Voyager spacecrafts are good candidates for this procedure.

If the transmitted ramp is of the form:

$$f(t) = \begin{cases} a't + f_0 & t'_0 \leqslant t < t'_1 \\ -a't + b' & t'_1 \leqslant t < t'_2 \\ a't + c' & t'_2 \leqslant t < t'_3 \\ f_0 & t'_3 \leqslant t \end{cases}$$

where:

a' = magnitude of the rate of change of frequency

f_0 = initial frequency

and b' and c' are constants, the form of the doppler residuals when the ramp is received at each DSS is:

$$\begin{aligned} R(t) = {} & R_0 && t < t_0 && \\ & a_1 t + b_1 && t_0 \leqslant t < t_1 && (1) \\ & a_2 t + b_2 && t_1 \leqslant t < t_2 && (2) \\ & a_3 t + b_3 && t_2 \leqslant t < t_3 && (3) \\ & R_1 && t_3 < t && \end{aligned}$$

For a symmetrical sawtooth frequency ramp (and $R_0 = R_1$), $a_1 = -a_2 = a_3$.

III. Data Analysis

Analysis of the data proceeds as follows: A least squares linear fit is made to $R(t)$ for each segment (indicated 1, 2, 3). For the fit, a total of four points are not used – these are the first and last points of each segment. For each segment 1, 2 and 3, a RMS deviation is computed. For an equation of the form:

$$R(t) = at + b$$

$$\sigma_R = \sqrt{\frac{\sum_{i=1}^{n} (\Delta R_i)^2}{n}}$$

where ΔR_i is the difference between the actual R and the fit R at t_i, and n is the number of data points obtained (see Figure 1).

The uncertainty of the coefficients a and b is given by:

$$\sigma_a = \sqrt{\frac{n}{n \sum_{i=1}^{n} (t_i)^2 - \left(\sum_{i=1}^{n} t_i \right)^2}} \ \sigma_R$$

$$\sigma_b = \sqrt{\frac{\sum_{i=1}^{n} (t_i^2)}{n \sum_{i=1}^{n} (t_i)^2 - \left(\sum_{i=1}^{n} t_i \right)^2}} \ \sigma_R$$

The points of interest are the times at which the peaks occur. These are:

$$t_1^* = \frac{b_1 - b_2}{a_1 - a_2}$$

and

$$t_2^* = \frac{b_2 - b_3}{a_2 - a_3}$$

The accuracy with which these times are known is given by:

$$\sigma_{t_1^*} = \left\{ \left(\sigma^2_{b_1} + \sigma^2_{b_2}\right) \left(\frac{1}{a_1 - a_2}\right)^2 + \left(\frac{b_2 - b_1}{(a_1 - a_2)^2}\right)^2 \left(\sigma^2_{a_1} + \sigma^2_{a_2}\right) \right\}^{1/2}$$

and a similar equation for $\sigma_{t_2^*}$. The equation can be put in a form containing σ_R and n (the number of data points for each segment) by approximating the sums in the σ_a and σ_b equations by integrals

$$\sum t_i \to \int t\,dt; \quad \sum t_i^2 \to \int t^2\,dt.$$

The result – for a sawtooth frequency ramp with a 20 minute period – is:

$$\sigma_{t_1^*} \cong \frac{\sigma_R}{(a_1 - a_2)\,n^{1/2}} \left\{ \left(\frac{3.67}{n}\right)^2 + (2.45)^2 \left(\frac{b_1 - b_2}{a_1 - a_2}\right)^2 \right\}^{1/2}$$

As can be seen from the equation, a large slope will reduce the uncertainty in t_1^*. Thus, using X-band will improve the accuracy of t^* by a factor of about 4.8 over S-band. Although choosing large n (short sample times) will increase the σ_R somewhat, since σ_t^* is proportional to $n^{-1/2}$, large n will reduce the uncertainty of t^* (approximately as $n^{-0.2}$). For example, 1 second data will reduce the uncertainty in t^* by a factor of about 1.6 over 10 second data.

For large n, the uncertainty in t^* is approximately:

$$\sigma_{t_1^*} \cong \frac{2.45\,|b_1 - b_2|}{(a_1 - a_2)^2\,n^{1/2}}\,\sigma_R$$

Thus, minimizing b_i is also important. No attempt was made below to minimize b_1, b_2 or b_3, but this is easily done.

I have gone through this analysis using ramp ranging data for Pioneer 10, taken at DSS 14 on DOY 209 (1977). The exciter VCO was increased at a rate of 2.083 Hz/sec. (This rate can be increased somewhat in future tests, without fear of losing lock on the ramp's return.) The received S-band slope is

$$\left(\frac{240}{221} \times 48 \times 2.083\right) + C_0 \cong 108.6.$$

The constant C_0 is a term involving the spacecraft motion. The results of the analysis are shown in Table 1, below, for 10 second data, and Table 2 shows the uncertainty of the time of the peaks.

For this day, the "Doppler Noise" (computed for 60 second data) was 0.007. This is high, noise levels of 0.003 are the rule and values slightly lower can be obtained using a Hydrogen maser. Thus, an improvement by a factor of 2 is expected for σ_t^* for standard data. This translates to specification of the event (the peak) to 0.49 μs at each station (1 second X-band data) resulting in a clock offset measurement accuracy of 0.7 μs (excluding other correction uncertainties).

All of the above calculations may be incorporated into the RTM – including a check of the doppler residuals to automatically activate the subroutine when a ramp occurs. What remains then is to apply corrections to the measured clock offset for 1) the difference in signal path length (divided by C), 2) the difference in electronic delays, and 3) the difference in atmospheric delays.

As mentioned previously, this method is compatible with VLBI. Several schemes can be used to correlate the data, and these are under investigation.

The advantage of this system will be a reduction of data processing time with considerable cost savings rather than an improvement of accuracy over the currently proposed VLBI clock sync method. This is possible because the spacecraft can be considered a coherent point source – thus, circumventing some correlation problems associated with the transverse coherence length of stellar radio sources. (This will allow similar accuracies with less data.) The signal level will be high and the time change of the frequency can be computed before the fact. This last point will allow the use of more narrow receiver bandwidths, driven by programmable local oscillators.

Table 1. Ten second data

Region	a_i (Hz/Sec)	b_i (Hz)	σ_R (X 10^{-3})	σ_a (X 10^{-6})	σ_b (X 10^{-4})
1	−108.60002	329.0453	7.59	5.06	0.835
2	108.60002	-65484.6439	9.61	0.436	4.71
3	−108.60002	130636.9870	10.02	6.68	70.4

Table 2. Uncertainty of the time of the peaks

Peak	σ_t^* (10 sec. S-band)	σ_t^* (1 sec. S-band)	σ_t^* (1 sec. X-band)
t_1^*	7.42 μs	4.67 μs	0.97 μs
t_2^*	33.8 μs	21.3 μs	4.4 μs

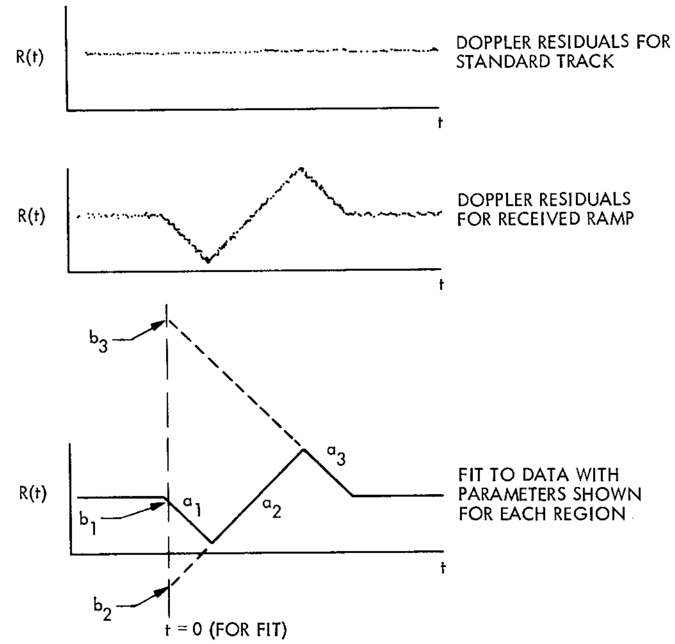

Fig. 1. Typical doppler residual plots