Doppler Phase-Noise Measurement Using Mean-Sweep Techniques

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This paper describes an investigation leading to techniques to reduce the error present in the DSN station measurement of doppler phase-noise variance and standard deviation. The error source of concern is the doppler counter resolver, which sorts continuous phase data into finite group intervals. The intervals are 3.5 deg (1 MHz bias) or 18 deg (5 MHz bias) wide.

Counter operation is covered to define parameters, and then appropriately limited model functions for the variance, its estimate, and the error are stated. The model introduces linear and sinusoidal mean-sweep functions to investigate their effect on the error.

Machine-programming of the model yields results which indicate that mean-sweep significantly reduces the measurement error when phase-noise standard deviation is low (less than 2 deg at 1 MHz bias, or 10 deg at 5 MHz bias). Linear sweep is the most accurate technique, but sinusoidal sweep is recommended as the more feasible; errors with the latter do not exceed 0.15 deg rms (1.0 MHz bias) in the region of interest.

Finally, the recommended parameters and the expected residual error are stated for use in test program configuration and data reduction; the error is asymptotic to a universal constant variance offset which can be appropriately subtracted to compensate for the group interval width at all noise levels.

I. Introduction

Present doppler phase-noise measurement techniques yield reasonably accurate estimates when the noise standard deviation (sigma) exceeds 2 deg rms (1.0 MHz bias) or 10 deg (5.0 MHz bias). However, when sigma is smaller than these values, the estimate obtained is increasingly variable with certain initial conditions, becoming practically meaningless below 1.0 deg (1.0 MHz bias) or 5.0 deg (5.0 MHz bias). Sigma levels within these regions are encountered in strong-signal coherent S-band test modes and X-band operations and test procedures.

An initial condition of interest in this region arises from the finite time-resolution of the doppler counter resolver. The resolver estimates the time-position of waveform zero-crossings, which form a continuously distributed set when random noise is present. The mean value of this set is, however, stationary with respect to adjacent resolver pulses when these pulses are time-coherent with the bias period, the usual test case. The time difference between the mean-value zero-crossing and the adjacent (earlier) resolver pulse, its estimate, is an error variable with initial conditions and cannot normally be predicted.

As this initial offset varies, the final estimate of sigma varies. When sigma is small, this variation is too large to yield meaningful measures with confidence. However, if this initial offset is intentionally varied by a known "mean-sweep" function during data collection, the error will tend to average, and the total possible variation will presumably decrease. The following discussion investigates this possibility in detail by modeling the resolver behavior during application of various sweep forms.

II. Doppler Counter Operation and Measurement Model

Doppler phase-noise measurement in the DSN stations is done by processing a set of doppler counter samples. Reduction of this data set yields an estimate of random phase-noise variance σ_ϕ^2 and/or standard deviation σ_ϕ. The latter is simply the rms value of the noise.

The variance and standard deviation depend on signal strength, mode of operation, system temperature, system noise, and many other factors. Doppler noise standard deviation is as low as 1 or 2 deg or less under strong-signal coherent conditions, when system noise is minimized.

The actual noise data extracted from the counter samples is a residual on the order of 1×10^{-8} of the phase accumulation between samples. The total differential phase between each sample is primarily the counter bias phase-accumulation over the sample period τ. The accumulation is normally between 10^5 cycles and 10^8 cycles, depending on the period τ and the bias frequency B. At the end of each sample period, the accumulated total cycle count (since reset) is read out as an integer. The readout is not the differential phase; the counter acts as a continuous digital integrator, summing these differentials. In addition, the counter is mechanized to obtain meaningful residual data by means of a resolver that measures (each sample period) the time position of the final waveform (positive-going) zero crossing of the period, within a resolution of 10 ns ($\Delta\tau$). The resolver tabulates the number of 10-ns intervals, an integer K. This integer is also read out and understood as an estimate ($K\Delta\tau$) of the true resolver period ΔT between the timing pulse that ended the sample period (t_p) and the time of the final positive-going zero crossing (t_z). The period $K\Delta\tau$ is not precisely equal to ΔT, and thus is not correct, for the tabulation is stopped at t_z, which normally occurs at some point between discrete resolver increments $K\Delta\tau$ and $(K+1)\,\Delta\tau$.

Expressed quantitatively,

$$\begin{aligned} t_z &= t_p + \Delta T \\ &= t_p + K\Delta\tau + \delta T \\ &\qquad 0 \leq \delta T \leq \Delta\tau \end{aligned} \tag{1}$$

where

ΔT = true time increment from t_p to the zero crossing t_z

$\Delta\tau$ = fixed resolver time increment, 10 ns

δT = error of $K\Delta\tau$ in measuring ΔT

K = resolver increment index or count

The quantity of interest is the residual phase ϕ_R, which is the (accumulated) difference between the true phase and the nominal phase due to bias alone. This can be expressed as a frequency integral yielding the integrated integer I described above:

$$\begin{aligned} I &= \int_{t_0}^{t_p+\Delta T} [B - \dot{\phi}_R(t)]\,dt \\ &= B[(t_p - t_0) + \Delta T] - \int_0^{t_p+\Delta T} \dot{\phi}_R(t)\,dt \\ &= B[(t_p - t_0) + \Delta T] - \phi_R \end{aligned}$$

where t_0 = time of first zero crossing after reset.

The nominal, due to bias alone, is

$$I_0 = \int_{t_0}^{t_p+\Delta T_0} B\,dt = B[(t_p - t_0) + \Delta T_0]$$

where t_0 is repeated; the start time is theoretically irrelevant. Or

$$(I - I_0) = \Delta I = B[\Delta T - \Delta T_0] - \phi_R$$

$$\phi_R \equiv \int_{t_0}^{t_z} \dot{\phi}_R(t)\, dt$$

Although actual mechanization restricts ΔT to a range of values, with small differences of ΔI completing the relationship, it simplifies the model, without loss of generality, to assume I and I_0 equal, with ΔT having complete freedom. This lead to

$$\begin{aligned}\phi_R &= B[\Delta T - \Delta T_0] \\ &= B[K\Delta\tau - K_0\Delta\tau] + B[\delta T - \delta T_0]\end{aligned} \tag{2}$$

By similar development, the estimate of ϕ_R, $\hat{\phi}_R$ can be shown to be

$$\begin{aligned}\hat{\phi}_R &= B[(K - K_0)\,\Delta\tau] \\ &= \phi_R - (\delta\phi - \delta\phi_0) + \int_{t_p + K\Delta\tau}^{t_p + K\Delta\tau + \delta T} \dot{\phi}_R(t)\, dt\end{aligned} \tag{3}$$

The integral is considered negligible; residual frequency components are too small to build up any appreciable value over the short time span. This yields (in parallel with ϕ_R)

$$\begin{aligned}\hat{\phi}_R &= B[K - K_0]\,\Delta\tau = \phi_R - B(\delta T - \delta T_0) \\ &= \phi_R - (\delta\phi - \delta\phi_0)\end{aligned}$$

These stable forms allow a convenient scaling; it is to set $K_0 = 0$ (shift the index origin to K_0) and simultaneously define the phase of the set of resolver levels ϕ_K by the notation

$$\phi_K = K\Delta\phi - \delta\phi_0 \tag{4}$$

where

$$\Delta\phi = B\Delta\tau = 3.6 \text{ deg } (1 \text{ MHz}) \text{ or } 18 \text{ deg } (5 \text{ MHz})$$

and

$\delta\phi = B\delta T_0 =$ the offset of mean ϕ_R from the nearest lower resolver level

These combine to

$$\begin{aligned}\phi_R &= \phi_K + \delta\phi \\ \hat{\phi}_R &= \phi_K + \delta\phi_0 = K\Delta\phi\end{aligned} \tag{5}$$

where

$$K = \cdots, -2, -1, 0, 1, 2, \cdots$$

$$-\infty < \phi_R < +\infty$$

The residual phase argument ϕ_R consists of all deviations from nominal bias phase. To further limit the discussion, consideration will be given to only two contributions; all others will be assumed negligible. The two are:

(1) Intentional, known, and deterministic deviations of the true mean from the nominal (bias) mean.

(2) Random phase noise, assumed stationary, gaussian, and normally distributed, with variance σ^2_ϕ.

With these, ϕ_R becomes

$$\phi_R = \bar{\phi} + \phi$$

where

$\bar{\phi} =$ intentional mean displacement of the entire ϕ distribution from nominal

$\phi =$ phase-noise displacement from $\bar{\phi}$ at any instant of measure

The phase quantities above and their time equivalents are diagrammed in Fig. 1, with a superimposed typical phase-noise probability density function. Note the actual measure ϕ at time t_z occurs between resolver levels ϕ_4 and ϕ_5. The (index-shifted) resolver readout would be the "K" index of ϕ_4, equivalent to $K\Delta\phi$ of $\hat{\phi}_R$, or "4," and any zero-crossing in the shaded area A_4 would yield this readout. The area A_4 thus represents the probability of the reading $4\Delta\phi$; other area probabilities would occur similarly.

After observation of Fig. 1, it becomes apparent that the various probability areas, such as A_4 shown, vary for given ϕ_K if the mean dimension $\bar{\phi}$ is varied, and that this will certainly not degrade the data if the mean variation is known.

The effect of sweeping is actually to vary the location of all $P(\phi)$ with respect to the $\{\phi_K\}$. Various initial $\delta\phi_0$ will thus obviously alter the result, since the ϕ_K are a function of $\delta\phi_0$. In fact, the purpose of mean-sweep is to eliminate or reduce the error due to arbitrary initial unknown $\delta\phi_0$ with a compensating, known, and averaging $\bar{\phi}$ function.

At the extreme, without sweep, if the entire $P(\phi)$ (of small variance) were (essentially) confined within one resolver level, then only a single reading would occur with high probability. The variance estimate would then be (nearly) zero. However, moving the mean would shift the distribution into various levels, and at least some data would seemingly be meaningful after mean-shift subtraction and other processing.

The effects of mean-shift by various deterministic functions is difficult to determine analytically because the $P(\phi)$ integral is transcendental. However, these effects can be readily modeled for machine computation, and this approach prevails in the remainder of the discussion.

III. Resolver Error Model Function and Mean-Sweep

Variance is estimated from data as the mean-square value of the data set, all data as deviations from the average value. During data collection, probabilities are not normally tabulated; they simply "occur" as variations in the number of data samples in the various class intervals. For finite data, the number of class intervals obtained is directly proportional to the resolution of the data samples with respect to the standard deviation.

Phase samples from the DSN doppler counter occur in class intervals $\Delta\phi$ wide, all samples in the class interval yielding the same value, $K\Delta\phi$, where K is the index of the interval.

For example, in Fig. 1, without differencing, the relative contribution of the shaded area (A_4) to the variance estimate would be (assuming an infinite sample population with $\bar{\phi} = 0$, and neglecting mean offset of the area)

$$\sigma_\phi^2(A_4) = (4\Delta\phi)^2 \int_{\phi_4}^{\phi_5} P(\phi)\, d\phi$$

$$4\Delta\phi = \phi_4 + \delta\phi_0 = \hat{\phi}_R \tag{6}$$

while the true contribution is

$$\sigma_\phi^2(A_4) = \int_{\phi_4}^{\phi_5} \phi^2 P(\phi)\, d\phi \tag{7}$$

Equations (6) and (7) show the essential error source; errors occur because, with sample data, a single value is used to represent the mean-square data of an entire region, the group interval. The true variance is defined such that the data are continuous and samples are "inside" the integral, squared for integration with the corresponding probability density coordinate. There is obviously some value of phase that could be subtracted from $4\Delta\phi$ of (6) (holding A_4 constant) for equivalence with (7); such values, however, would not be static, but would rather depend on ϕ_K. The "correction" is thus nonstatic and must be determined as a function of the areas, or, finally, of the variance estimate.

It is intuitive that the variance error will increase when the probability integral is large with respect to variance. When the limits are infinite, this integral is unity, and the entire estimate is a single measure, which is meaningless.

On the other hand, when the interval is (relatively) small with respect to variance, the estimate and true values asymtotically approach, for the relative change in argument across each interval is small.

If mean-sweep is introduced, the class interval associated with a given estimate, such as $4\Delta\phi$, shifts in location, while the actual estimated angle $(\hat{\phi}_R - \bar{\phi})$ takes on a variety of values. A "weighted average" contribution to variance thus occurs for each $\phi_K = K\Delta\phi - \delta\phi$. The model for this estimate and its true counterpart can be expressed in some detail by (including a nonzero initial mean)

$$\sigma_\phi^2 = \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [\bar{\phi} - \phi(t)]^2 P[\bar{\phi} - \phi(t)]\, d\phi\, dt \right\} - (MN)^2 \tag{8}$$

$$= \left\{ \int_{-\infty}^{+\infty} \left[\sum_{K=-\infty}^{\infty} \int_{\phi_K - \bar{\phi}(t)}^{\phi_K - \bar{\phi}(t) + \Delta\phi} \phi^2 P(\phi)\, d\phi \right] dt \right\} - (MN)^2 \tag{9}$$

$$\hat{\sigma}_\phi^2 = \left\{ \frac{\tau}{T} \sum_{M=1}^{\frac{T}{\tau}} \sum_{K=-K_L}^{K_L} [K\Delta\phi - \bar{\phi}(M\tau)]^2 \int_{\phi_K - \bar{\phi}(M\tau)}^{\phi_K - \bar{\phi}(M\tau) + \Delta\phi} P(\phi)\, d\phi \right\} - (MN)^2 \tag{10}$$

σ_ϕ = phase noise variance

$\hat{\sigma}_\phi$ = estimate of σ_ϕ during typical measurement

ϕ = phase noise deviation from mean (continuous variable)

$P(\phi)$ = probability density function of argument; commonly "normal" or "gaussian"

K = index of resolver phase levels, modulo B_T

ϕ_K = expression of resolver levels as incremental phase deviations, $\Delta\phi$ wide, from ϕ_0, the phase level nearest the nominal mean

$\Delta\phi$ = ϕ_K increment, 3.6 deg (1.0 MHz) or 18 deg (5.0 MHz)

MN = $P(\phi)$ mean value, exclusive of $\overline{\phi}$

τ = sample interval, usually 1.0 or 0.1 s

T = total measurement period, 10 s to several minutes

$-K_L, K_L$ = lower and upper index limits for class intervals of obtained data

$\overline{\phi}(t), \overline{\phi}(M\tau)$ = deviations of mean value of ϕ distribution, at time t, from nominal value. At times $M\tau$, the nominal (null) value is the modulo B_T bias waveform phase within the resolver; $\phi_0 + \delta\phi_0 = 0$; $\overline{\phi}(t)$, $\overline{\phi}(M\tau)$ are presumed intentional and known

Note that (9) differs from (10) in that (10) is not only discrete in the mean-sweep function, but also the phase measures $\{K\Delta\phi - \overline{\phi}(M\tau)\}$ are again outside of the probability integral. Each set of measures therefore represents, as before, a class interval of probability given by the class interval integral. Conversely, (9) is exact, since ϕ^2 is within the integral.

Note that, in (10), many more class intervals are obtained because the mean-sweep function is not constant, but steps to various values. These intervals are *not* distinct but overlap; the error is not "removed," but rather "pseudo-averaged" to some difference value by sweeping.

Various undesirable effects mentioned earlier, but not evident in (10), lead to actual data reduction as a differencing of the sequential $\{K\Delta\phi\}$ obtained. Upon differencing, the defining expressions become somewhat intricate multiple sums and integrals:

$$\sigma_\phi^2 = \frac{1}{2}\left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} [\phi_2 - \phi_1 - \Delta M(t)]^2 \times P[\phi_2 - \overline{\phi}(t+\tau)]\, P[\phi_1 - \overline{\phi}(t)]\, d\phi_1 d\phi_2\, dt\right\} - \frac{(MN)^2}{2} \tag{11}$$

$$\hat{\sigma}_\phi^2 = \frac{1}{2}\left\{\frac{\tau}{T}\sum_{M=1}^{\frac{T}{\tau}}\sum_{K=-K_L}^{K_L}\sum_{N=-K_L}^{K_L}[N\Delta\phi - \Delta M(M\tau)]^2 \times \int_{\phi_K-\overline{\phi}(M\tau)}^{\phi_K-\overline{\phi}(M\tau)+\Delta\phi} P(\phi_2)\,d\phi_2 \times \int_{\phi_K-\overline{\phi}(M\tau-\tau)}^{\phi_K-\overline{\phi}(M\tau)+\Delta\phi} P(\phi_1)\,d\phi_1\right\} - \frac{(MN)^2}{2} \tag{12}$$

where

ΔM = intentional change in mean phase during sample period

ϕ_1, ϕ_2 = two arbitrary sequential phase measures, taken τ seconds apart

Note that the bias mean offset from ϕ_0, $\delta\phi_0$, contained in all ϕ_K, is *not* normally known, so any mean-sweep function will necessarily have an arbitrary zero reference, whose effect may or may not be finally significant. Also, if $P(\phi)$ is a stationary function about the mean, MN will be zero upon differencing, and may be dropped.

The factor "½" in each expression results because the differenced distribution variance is automatically twice as large as the variance of the primary distribution. This is well established and will not be covered here.

For actual machine programming, $P(\phi)$ was considered gaussian and therefore stationary. The error-function subroutine, adjusted for gaussian form, was used to evaluate the integrals. Also, (11) could feasibly be expressed as a single integral since the difference between variates in a gaussian distribution is itself a gaussian distribution with doubled variance. Sweep is obviously irrelevant since it would simply "subtract out" as an additive contribution. However, to avoid undue complication, (12) is easier to handle in the double-integral form, since mean-differences cause probability changes between samples. The single-integral form was modeled, but it was more complicated than (11), so it was dropped in favor of the expression as stated.

Data on available DSN hardware indicated that sinusoidal modulation index could be set within a tolerance of $\pm 5\%$. With this in mind, a simulation program for (12) was automated. The program routine to calculate estimated sigma was:

$$\hat{\sigma}_\phi^2 = \left\{ \frac{\tau}{8T} \sum_{M=1}^{T/\tau} \sum_{K=-K_L}^{K_L} \sum_{N=-K_L}^{K_L} (N\Delta\phi)^2 \left[\mathrm{erf}(\theta_2) - \mathrm{erf}(\theta_1)\right] \times \left[\mathrm{erf}(\theta_4) - \mathrm{erf}(\theta_3)\right] - \frac{\tau}{2T} \sum_{M=1}^{T/\tau} \left[\hat{\Delta}M(M\tau)\right]^2 \right\}^{1/2}$$

$$\theta_1 = \frac{[K\Delta\phi - \bar{\phi}(M\tau - \tau) - \delta\phi_0]}{\sqrt{2}\,\sigma_\phi}$$

$$\theta_2 = \theta_1 + \Delta\phi/\sqrt{2}\,\sigma_\phi$$

$$\theta_3 = \frac{[(K+N)\,\Delta\phi - \bar{\phi}(M\tau) - \delta\phi]}{\sqrt{2}\,\sigma_\phi}$$

$$\theta_4 = \theta_3 + \Delta\phi/\sqrt{2}\,\sigma_\phi$$

$$K_L = \mathrm{int}\,[6\,\sigma_\phi/\Delta\phi] + 1$$

$$\bar{\phi}(M\tau) = \begin{cases} 0 \text{ (no sweep)} \\ (M\tau/T)\,\Delta\phi \text{ (linear sweep)} \\ \phi_M \sin[2\pi\, M\tau/T] \text{ (sinusoidal sweep)} \end{cases}$$

$$\hat{\Delta}M(M\tau) = \begin{cases} 0 \text{ (no sweep)} \\ (\tau/T)\,\Delta\phi \text{ (linear sweep; no appreciable error)} \\ \hat{\phi}_M \{\sin[2\pi\, M\tau/T] - \sin[2\pi\,(M\tau - \tau)/T]\} \end{cases}$$

$$\hat{\phi}_M = \text{estimated modulation index, deg}$$

$$\phi_M = \text{true modulation index, deg}$$

$$\delta\phi_0 = \text{initial offset}, 0 < \delta\phi_0 < \Delta\phi \qquad (13)$$

Note that ϕ_M, the true (but unknown) modulation index, appears in the error function integral, while $\hat{\phi}_M$, the data reduction value, appears only in the rms correction due to sinusoidal sweep.[1] Since the resolver mean offset $\delta\phi_0$ is modulo $\Delta\phi$, it is evident that limiting $\delta\phi_0$ to a spread of $\Delta\phi$ will cover all variation. An initial investigation with machine programming of (13) alone indicated that the error variation with modulation index ϕ_M tended to increase when ϕ_M was greater than $\Delta\phi$ or less than about $\Delta\phi/3$. Therefore, detailed investigation was limited to this range.

An error measure was also in the program along with a mean error (offset) calculation. These measures were

$$\theta_0(\hat{\sigma}_\phi) = \frac{1}{m+n} \sum_{a=1}^{m} \sum_{b=1}^{n} \hat{\sigma}_\phi(\phi_{M_a}, \delta\phi_{0_b})$$

$$\epsilon^0_{\mathrm{rms}}(\hat{\sigma}_\phi) = \left[\frac{1}{m+n} \sum_{a=1}^{m} \sum_{b=1}^{n} [\hat{\sigma}_\phi(\phi_{M_a}, \delta\phi_{0_b}) - \theta_0(\hat{\sigma}_\phi)]^2 \right]^{1/2} \qquad (14)$$

[1]In (13), this correction has been separated from the main argument for independent evaluation. This is possible upon differencing; the cross-correlation is zero.

Results used had the tolerance spread

$$m = 3 \text{ for } \phi_M = \hat{\phi}_M, \hat{\phi}_M + 5\%, \hat{\phi}_M - 5\%$$

$$n = 4 \text{ for } \delta\phi_0 = 0, 0.9, 1.8, 2.7^\circ \qquad (15)$$

where

$\theta_0(\hat{\sigma}_\phi)$ = the mean error, or average offset, between given sigma and its estimates

$\epsilon^0_{\mathrm{rms}}(\hat{\sigma}_\phi)$ = the rms value of the variation in $\hat{\sigma}_\phi$ for given σ_ϕ due to (uncontrollable) tolerances in ϕ_M and initial values of $\delta\phi_0$.

The error statistic $\epsilon^0_{\mathrm{rms}}$ is not a random variate but is bounded by the tolerance excursions and their combined effects.

Equations (14) and (15) complete the quantitative description of the model program. The program inputs were sigma (σ_ϕ), sample interval (τ), sweep repetition period (T), and estimated modulation index ($\hat{\phi}_M$). The fundamental sweep frequency was obviously τ/T Hz. To avoid the region where autocorrelation effects would possibly occur in practice, τ was chosen as 1.0 *s*. The period T was then set arbitrarily at 10.0 s. This yields the lowest practical sinusoidal sweep frequency (0.1 Hz) available at the DSN sites. A lower frequency would yield a higher sweep resolution, obviously desirable, but not presently feasible.

Sigma values from 0.4 to 3.0 deg were processed as the modulation index was varied over the stated range. Also, highly resolved linear sweep ($\tau/T = 0.01$) as well as no-sweep conditions were cycled, and the various data were collected for analysis.

IV. Mean-Sweep Effects and Error Analysis

The error statistic $\epsilon^0_{\mathrm{rms}}$ represents a medium value of the variation in the estimate of the error, as to be expected when measuring doppler phase-noise sigma while sweeping. With the highly resolved linear sweep, this error was all but undetectable. Such sweep would be preferable to all other forms, except that it is difficult to mechanize during coherent testing.

At the other extreme, the error with "no-sweep" was larger than σ_ϕ itself at the lower values (below 0.5 or 2.5 deg) and rendered the measure questionable until σ_ϕ was greater than 2.0 deg (or 10 deg). For sigma greater than these levels, sweep made little difference; the error was negligible.

With sinusoidal sweep, the error in the region of interest was less than one-fourth the no-sweep level for all modulation indices programmed. Within this bound, however, the relative error was very sensitive to the actual $\hat{\phi}_M$. The $\hat{\phi}_M$ between 1 and 3 deg was tried, and the optimum value appears to be about 1.5 deg. The error distribution with various $\hat{\phi}_M$ was very erratic within these small limits, but finally insignificant because of their low absolute level. Within the measurable range, the rms error never exceeded 0.15 deg (1.0 MHz) while sweeping ($\sigma_\phi = 0.4$ deg), and this model level, in practice, will undoubtedly be masked by other system contributions.

In order to facilitate easy mechanization, a modulation index of 2.0 deg was selected for test use, as compatible with the DSN Command Modulator Assembly (CMA) equipment.

A typical sinusoidal-sweep error distribution of $\hat{\sigma}_\phi$, yielding $\epsilon^0_{\mathrm{rms}}$ and θ_0, is shown in Fig. 2 as a function of $\delta_{\phi 0}$ and $\hat{\phi}_M$. The figure, for a σ_ϕ of 0.6 deg, clearly shows the pseudo-sinusoidal nature of the error; peak error is less than 1.5 times the rms value.

Composite error results described above are shown in Fig. 3. Note the large error reduction in the small-sigma range by sinusoidal sweeping as opposed to the existing no-sweep condition. Also note the convergence of all data at about 2 deg (or 10 deg); sweeping to improve statistics above these levels is, as noted, superfluous.

The statistic θ_0 in (14) is very interesting. For $\hat{\phi}_M$ of 2 deg, the selected value θ_0 varies from an asymptote of about 1.040 deg (1.0 MHz) at the higher levels of σ_ϕ down to 1.023 deg at the minimum measurable level, an insignificant change of only 0.017 deg across the entire field. It obviously differs little from a constant rms offset, apparently related to the resolver increment $\Delta\phi$.

With a little manipulation, the limiting value of this offset can be shown to have a well-known quantization error value (omitting intermediate steps):

$$\lim_{\sigma \to \infty} (\theta_0)^2 = \left[\lim_{\sigma \to \infty} \sum_{K=-\infty}^{\infty} \frac{1}{\Delta\phi} \int_{(K-1/2)\Delta\phi}^{(K+1/2)\Delta\phi} \phi^2 P(\phi,\sigma)\, d\phi - \sigma_\phi^2 \right]$$

$$= \frac{\Delta\phi^2}{12} \qquad (16)$$

For $\Delta\phi = 3.6$ deg (1.0 MHz), this is 1.039 deg, in excellent agreement with the data. Since the variation in θ_0 with σ_ϕ is so small, the easiest correction to data is to simply subtract the θ_0 limit as an rms level:

$$\hat{\sigma}_{\phi\,\mathrm{corrected}} = \hat{\sigma}_c = \left[\hat{\sigma}^2 - \frac{\Delta\phi^2}{12} \right]^{1/2} \qquad (17)$$

This is the expression recommended for DSN doppler noise correction (with or without sweep) to account for the finite resolver group interval error, as desired.

The final distribution of $\hat{\sigma}_c$ with (17) and sinusoidal sweep with $\phi_M = 2$ deg applied (one-sigma error limits) for no sweep is shown in Fig. 4; $\hat{\sigma}_\phi$ (corrected) is the expected range of values during DSN data reduction when the specified sweep is used.

V. Conclusions

The primary purpose of doppler phase-noise measurement is to estimate the noise content of the doppler data obtained operationally and to assure that this noise is within specified bounds. This discussion has concentrated on the low-noise cases.

Operational data occurs with natural mean-sweep, often nearly linear, as a result of relative ground–spacecraft motion. Testing without sweep thus cannot accurately predict the operational noise in the strong-signal region; the estimate (as analyzed here) will normally be too variable.

In the S-band region (1.0-MHz bias) the no-sweep error is small except for somewhat unusual conditions, normally without operational application. Low-noise occurs, for example, when testing under fully coherent conditions, resulting in cancellation of exciter noise within the doppler extractor. However, when such conditions are encountered, as during trouble-shooting, mean-sweep will improve the residual measure (of the receiver) by reducing adverse resolver effects. Such effects have been encountered in practice.

In the X-band region (5.0-MHz bias), system noise is comparable to or less than resolver error without sweep. Measure of system noise, as it occurs operationally, appears to require a correction such as that of the sweep technique. It will remove a significant variability present during test, but normally absent during active tracking.

Sweep with the stated parameters reduces the rms of the test error by as much as 4-to-1 in the applicable region.

Independent of sweep effects, there exists a resolver quantization offset error, unavoidable because of the incremental pulse spacing. This will be present on all data, operational and test, and is about 1 deg at S-band (1.0 MHz) or 5 deg at X-band (5.0 MHz). If system noise or operational data outside of counter effects are to be measured, this offset must be removed. However, if an operational noise estimate is to be made, this may or may not be included as appropriate. For testing without this effect, it is recommended that the offset be extracted from the calculated estimate of counter input noise.

To summarize, when the bias is fixed and coherent, a sinusoidal sweep (by exciter phase modulation), with amplitude 2 deg and frequency 0.1 Hz, will adequately simulate natural operational sweep. This will reduce counter error sufficiently that meaningful results at sigma levels below 2 deg (1 MHz) and 10 deg (5 MHz) can be obtained. An additional fixed resolver quantization error of 1.039 deg (S-band) or 5.19 deg (X-band) may be subtracted (variance subtraction) as desired; it is a "built in" error on all doppler counter measures.

Definition of Symbols
(all time notation in seconds; all phase notation in cycles or degrees)

A_K — Kth area of probability density function between resolver levels

a — index (subscript) for modulation index

B — bias frequency, 1.0 to 5.0 MHz

b — index (subscript) for initial mean offset angle $\delta\phi_0$

$\epsilon^0_{\rm rms}$ — root-mean-square error of given sigma measure

K — resolver level index, resolver level phase subscript

K_0 — nominal K nearest (below) bias-mean zero-crossing

K_L — summation limit for K (absolute)

M — discrete measurement index and subscript

m — summation limit, modulation index

N — differential K on successive measures

n — summation limit, initial offset phase

$P(\phi)$ — probability density function notation

t — time, general notation, s

t_p — time of measurement pulse

t_z — time of first positive-going zero-crossing after t_p

T — total measurement period; period of periodic sweep

ΔT — actual time from t_p to t_z in resolver

ΔT_0 — mean value of ΔT, due to bias alone

$\Delta\tau$ — resolver time increment, 10 ns

$\Delta\phi$ — resolver phase increment $B\Delta\tau = 3.6$ deg (1.0 MHz), 18 deg (5.0 MHz)

ΔM — change in $\bar{\phi}$ between successive measures

δT — resolver time error in measurement of ΔT

δT_0 — mean value of δT, due to bias alone

$\delta\phi$ — resolver phase error in measurement of ϕ_R

$\delta\phi_0$ — mean value of $\delta\phi$, due to bias alone

ϕ — phase-noise displacement, general

ϕ_K — resolver level phase notation set

ϕ_R — total residual phase displacement from bias mean

$\hat{\phi}_R$ — $K\Delta\phi$, the resolver estimate of ϕ_R

ϕ_M — phase modulation index during sinusoidal sweep

$\hat{\phi}_M$ — operational estimate of ϕ_M

$\bar{\phi}$ — intentional mean displacement of $P(\phi)$ from bias mean; sweep function value

θ_0 — mean value of rms error between σ_ϕ and $\hat{\sigma}_\phi$, given σ_ϕ

τ — sampling interval, nominally 1 s

σ_ϕ — sigma (standard deviation) of ϕ

$\hat{\sigma}_\phi$ — estimate of σ_ϕ using counter data

$\hat{\sigma}_c$ — corrected value of $\hat{\sigma}_\phi$, based on θ_0

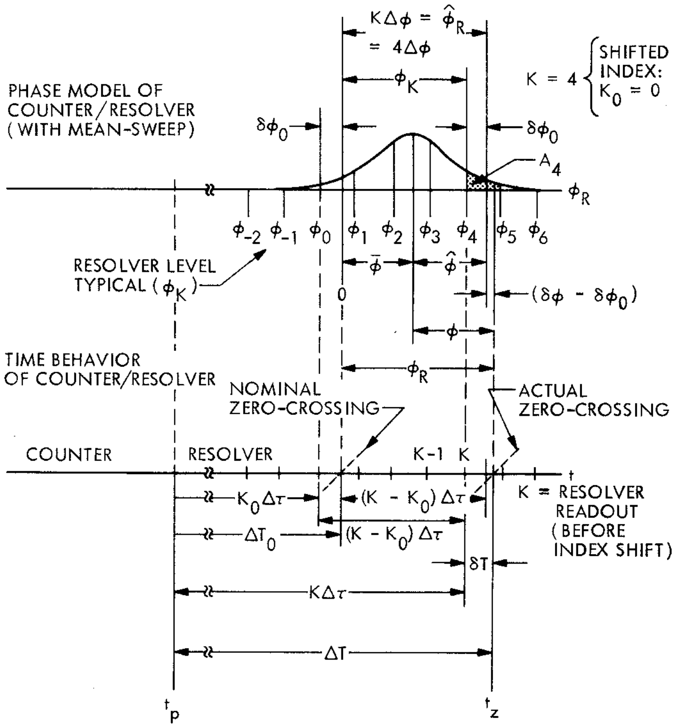

Fig. 1. Doppler counter resolver time/phase relationships

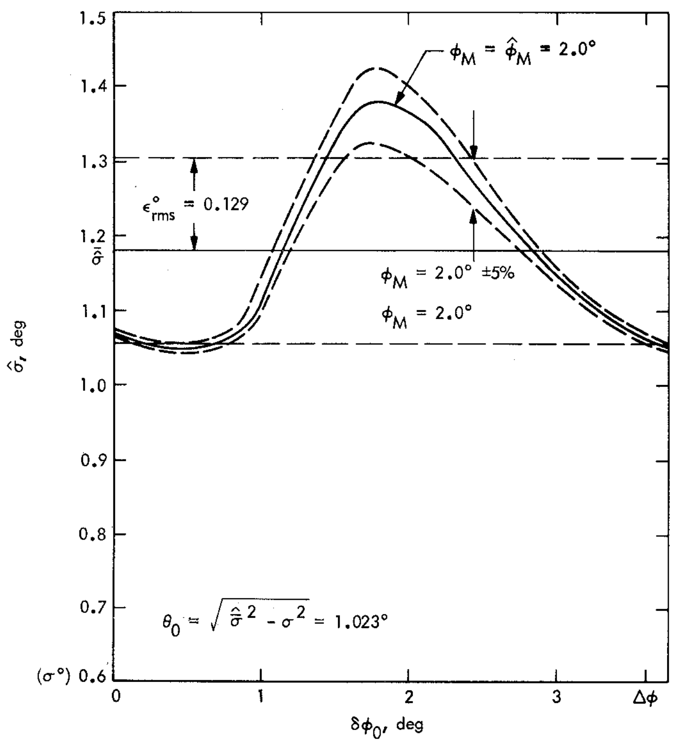

Fig. 2. Typical variation in noise sigma estimate $\hat{\sigma}°$ as a function of resolver mean offset ϕ_0 and modulation index tolerance $\phi_M \pm 5\%$; $\sigma = 0.6$ deg

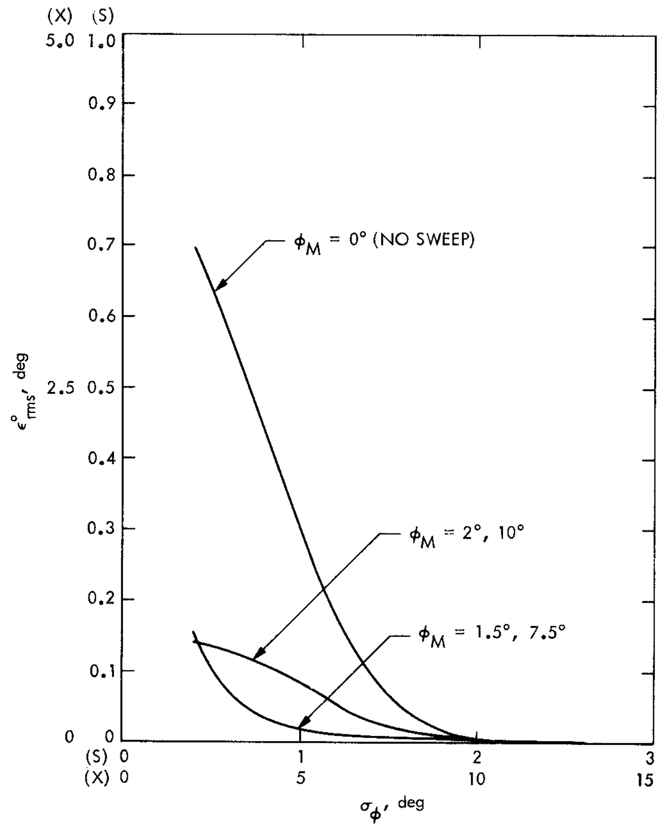

Fig. 3. Phase-noise measurement error as a function of sinusoidal mean-sweep for various modulation indices

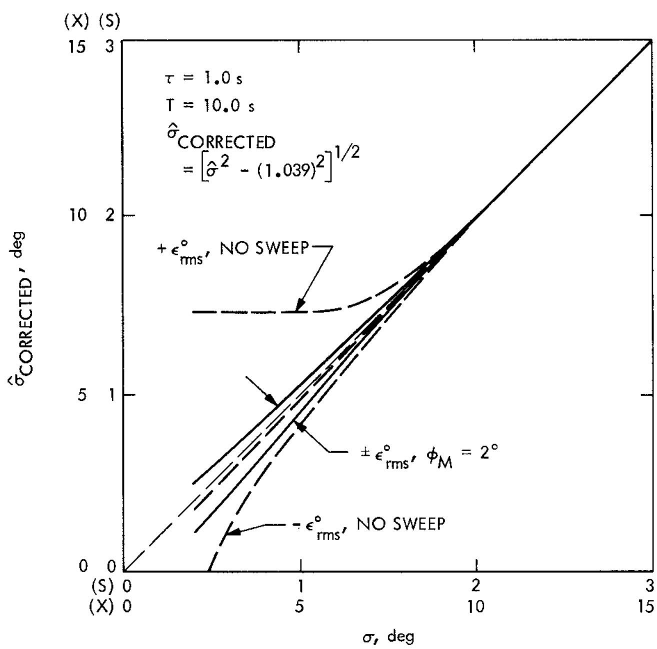

Fig. 4. Corrected sigma estimate vs sigma: no-sweep and sinusoidal sweep (mod index = 2°)